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Abstract

This is the final report of the research program "Compatibility of Vehicles Within a Platoon (MOU-156)." This research area is continuing under a different, contract and thus, although the work presented herein is complete unto itself, the overall research goals laid out at the beginning of the program will not be attained until the conclusion of the entire related research effort.

The material presented in this report provides a description of a simple dynamical model that can be used for determining the physical interaction of vehicles in a collision scenario. Such a model is of use in platoon simulations for which vehicle-to-vehicle impacts occur. Due to the restricted amount of real world data, only a very basic model can reliably be presented.

A source of real world collision data is identified, and a procedure for determining model parameters to match the data is given. Model implementation issues are discussed, and the results of the completed model are presented.

keywords: performance, platooning, safety, AHS, collision

Nomenclature

c_b	Bumper damping constant (Ns/m).
$c_{b,f}$	Linear damping constant in front vehicle (Ns/m).
$c_{b,r}$	Linear damping constant in rear vehicle (Ns/m).
$C_{b,f}$	Maximum damping force in front vehicle bumper (N)
$C_{b,r}$	Maximum damping force in rear vehicle bumper (N).
F	Force (N).
F_{body}	Force applied to vehicle body (N).
F_f	Force acting on front car mass (N).
F_r	Force acting on rear car mass (N).
F_o	Vehicle body break out force (N).
I	Identity matrix.
k_b	Bumper spring constant (N/m).
k_c	Vehicle body spring constant (N/m).
$k_{c,f}$	Body stiffness in front vehicle (N/m).
$k_{c,r}$	Body stiffness in rear vehicle (N/m).
$k_{b,f}$	Bumper stiffness in front vehicle (N/m).
$k_{b,r}$	Bumper stiffness in rear vehicle (N/m).
L_b	Dead length of bumper (m).
L_c	Active length of car body (m).
m	Mass (kg).
p_f	Position of front vehicle mass (m).
p_r	Position of rear vehicle mass (m).
v	Velocity (m/s).
v_f	Velocity of front vehicle mass (m/s).
v_r	Velocity of rear vehicle mass (m/s).
w	Vehicle Width (m).

x_{crush}	Total crush of vehicle (m).
x_c	Residual crush in vehicle body (m).
x_b	Instantaneous bumper crush (m).
$x_{i,0}$	Linearized system operating point.
δ	Preload distance on car body spring (m).
δ^R	Scalar, real uncertainty.
δ^C	Scalar, complex uncertainty.
δx_i	Linearized system state variable.
$\Delta^C_{m\mathbf{x}n}$	Complex full block of size m by n.
$\mathbf{\Delta}$	Structured uncertainty set.
ΔV	Velocity Change (m/s).

1 Introduction

As the population in the United States increases, the number of vehicles traveling on U.S. roads will also increase. Many highways, especially urban highways, have already grown to their limit. In areas such as the 1-80 stretch from Richmond, CA to Emeryville, CA a major traffic corridor has reached its growth limit, yet is often clogged. With the San Francisco Bay to the west and developed properties to the east, the only way to increase the size of the road is to build an upper deck. In an earthquake prone area such as California, this is not an attractive alternative.

To deal with the road overcrowding problems, options such as increased public transportation utilization or increased throughput of existing roads must be explored. In the United States citizens have become accustomed to the freedom provided by individualized transportation, thus are in general unaccepting of group transportation. One solution to the roadway crowding problem that allows users to retain the benefits of individualized transportation is the IVHS (Intelligent Vehicle Highway System).

The IVHS program seeks to increase the throughput of the current roads by equipping vehicles and roadways with control, sensing, and communication systems. The road/vehicle system would thus be "intelligent", and would allow more efficient usage of existing roadways. This increase in roadway efficiency would be realized by increasing vehicle speeds while drastically reducing the "safe" following distance between vehicles. Research by Hitchcock [1] shows that these changes could give automated lanes triple the vehicular flow of the same lanes under manual control.

Within the scope of IVHS there are several viable strategies for determining vehicle spacing. The California Program for Advanced Technologies for the Highway (PATH) currently favors the platoon architecture. This paradigm calls for vehicles to move in closely spaced strings, called platoons, with large spacings between the platoons. Under the multi-layered platoon architecture, vehicle control takes place in the platoon and regulation layers. [28]

To date, most platoon control research has focused on the development of control systems for platoons operating under nominal conditions. Relatively little attention has been paid to the performance of these control systems during emergency situations. Any system as complex as IVHS is subject to subsystem failures that can lead to emergency situations. In order to understand the behavior of control systems in such situations, one needs a vehicle collision model that can be used in platoon simulations. The intent of this paper is to provide such a model.

2 Problem Statement

In order to facilitate the development of viable platoon control systems for IVHS, the Dynamic Systems Laboratory at the University of California at Berkeley has undertaken the development of a one-dimensional Platoon Simulation Package (PSP). This is a Matlab based package that allows future expansion due to its highly modular design. The PSP requires a collision dynamics model (CDM) so that it can be used to predict platoon behavior in both nominal and collision scenarios.

To be of use to the control development community, the CDM must satisfy several requirements. The model must provide a reasonable representation of the physical deformation imposed upon a vehicle during collisions involving both high and low relative velocities (ΔV). It must also provide the other model subsystems of the PSP with the forces generated during a collision so that subsequent motions reflects the collision's occurrence. Finally, the model should be computationally unobtrusive, so that PSP performance is not adversely affected by the presence of collisions. The following section discusses several techniques that can be used to model vehicular collisions.

3 Model Types

3.1 Computationally Intensive Models

3.1.1 Finite Element Analysis

Finite Element Analysis (FEA) is a technique for modeling continuous structures without the use of partial differential equations. FEA approximates continuous structures with a series of discretized systems. As the number of discrete systems, known as finite elements, is increased the model better approximates the continuous system. Each of the finite elements is treated as having constant properties. In this manner, the continuous system can be transformed into a set of coupled ordinary differential equations (ODE's). Numerical solutions techniques for these types of problems are available.

Of all of the modeling techniques, FEA provides the most accurate vehicle deformation and collision force data. Unfortunately, FEA is not well suited to dynamic simulation. Because an FEA model contains a large set of coupled ODE's, it is computationally intensive to solve. Also, FEA provides solutions to static problems. Thus, a dynamic implementation of an FEA model would require the solution of a separate FEA problem at each time step. In addition, properly setting up an FEA model requires that the complete structure and material properties of the system be known. Thus, extensive data for the vehicles to be included in the simulation package is needed. For these reasons, FEA has been rejected as a modeling technique for our simulation studies.

3.1.2 Proprietary Models

Proprietary models have been developed for the express purpose of vehicle collision simulation. These packages use a variety of techniques to provide the user with collision data. Their use has been rejected because of their expense, computational requirements, and difficulty of integration into the PSP. Because of the many difficulties associated with proprietary models, they will not be used for collision modeling in the package.

3.2 First Principle Models

3.2.1 Impulse-Momentum

Impulse-Momentum (IM) models rely on the principle of conservation of momentum of a system. This is the traditional technique used for accident reconstruction [9], and has been implemented in a two-dimensional platoon simulation model by Moon [10]. Unfortunately, IM models have some drawbacks. In general, IM models only deal with two colliding bodies. The inclusion of bumpers is required in the CDM to handle low ΔV collisions. Because of this requirement, IM models are not well suited for use in the CDM. In addition, IM models also provide incomplete deformation information. The models presuppose a generalized coefficient of restitution that acts over small time intervals whereas a general simulation is better served by a force based interactions that can be charted over time. The difficulty with the small time assumption of IM collisions is significant. After the IM calculations have been performed, it is possible to determine the length of time that the collision would have taken. Unfortunately, if additional forces were introduced into the system during this time, possibly by another collision, the original calculation is invalidated. The complications introduced by IM models make them a non-optimal choice for implementation in the PSP.

3.2.2 Lumped Parameter Models

Lumped-Parameter (LP) models are the traditional approach to the modeling of dynamical systems. These models are based on Newton's Second Law of Motion

$$\frac{d}{dt}mv = \sum F_{applied}$$

or a form of it. The alternative, a continuum model, is too computationally intensive to allow its use. LP models have the benefit of being intuitive to design, and they are not computationally intensive. Models of this form were suggested by Tongue and Yang [11, 12, 13]. In addition, LP models are easy to fit to the behavior of a vehicle in a fixed barrier collision as presented by Strother et. al.[16]. If sufficiently efficient continuum models are developed then these can replace the more simplified lumped models being used herein. For the above reasons, the LP modeling technique will be used to develop the CDM for the PSP.

4 Database

After selection of the modeling technique to be used for the PSP, real world collision data was sought,. Data from actual vehicle collisions is required so that the model can be tuned to perform as the real system would. Two types of databases were considered, both dealing with actual vehicle collisions. The first type is built up of actual accident records. The most prevalent of these, the National Accident Sampling System, contains information on thousands of accidents collected over the span of many years. While this database contains a large amount of general information about vehicle deformation and accident survivability, it in not, suitable for modeling purposes due to the lack of precollision information. The second type of database considered consists of compilations of multiple crash tests. While these databases abound, only the National Highway Transportation Safety Administration (NHTSA) Research and Development Vehicle Crash Test Database [7] provided information on a variety of different vehicles with both front and rear collision information.

The NHTSA Database contains information on approximately 2100 controlled vehicle collisions staged by NHTSA and independent contractors. The tests were performed on a variety of vehicles in a variety of configurations from direct head on wall collisions to two vehicle offset collisions. The vehicles represent most domestic and many foreign auto makers, and encompass model years 1971 to the present.

For each test there are approximately 225 data records. These records are divided into five major categories, although all categories do not apply for all tests. The categories are: Test Configuration, Vehicle Data, Barrier Data, Occupant Data (test dummies), and Instrumentation Package data. For the collision dynamical model, the total data set was filtered so as to leave only vehicle data for cars of model years 1985 to the present which had been tested in both frontal and rear barrier collisions with direct impacts. After this process there remained data sets for 18 vehicles. The actual make and model of the remaining vehicles can be found in Table 1. From these data sets, a majority of the recorded categories were removed, leaving only information pertaining to the vehicles physical attributes and the test data. After this data was processed, the final data set contained the records shown in Table 2.

Table 1: NHTSA Database Vehicles

Make	Model	Model Year
Acura	Integra	1990
Acura	Legend	1988
Buick	Regal	1988
Chevrolet	Astro	1985
Chevrolet	Blazer	1985
Chevrolet	Blazer	1993
Chevrolet	Lumina	1990
Chevrolet	Spectrum	1985
Ford	Mustang	1994
Ford	Ranger	1993
Ford	Taurus	1986
Honda	Civic	1988
Hyundai	Pony Excel	1990
Mazda	323-Protege	1986
Subaru	DL	1985
Toyota	Celica	1986
Toyota	T100 Pickup	1993
Yugo	GV	1986

Table 2: Processed Database Records

Field	Units
Mass	kg
Wheelbase	m
Center of Gravity Position	m
Length	m
Front Width	m
Rear Width	m
Front Stiffness	N/m
Front Break-out Force	N
Rear Stiffness	N/m
Rear Break-out Force	N

5 Collision Dynamics Model

The collision dynamics model consists of five subsystems that are dynamically coupled. These systems include a bumper and body model for both the front and rear of the vehicle and a lumped vehicle mass. The system inputs are the forces applied to the front and rear bumpers. System outputs are the deflections in the bumpers and body sections, as well as the acceleration of the body mass. Figure 1 shows a schematic of the model. The model for the front and rear body systems is based on the model developed by Strother et. al. [16]. The structure of the bumper model was suggested by Tongue and Yang [11].

5.1 Bumper Model

In the United States, current law mandates that vehicle bumpers provide protection from permanent body damage in 5 MPH (8 KPH) collisions while sustaining minimal damage themselves. For the CDM to adequately handle these low energy collisions, a vehicle bumper model is required. Before discussing the specific bumper model used in the CDM, two assumptions must be made.

First, the bumper model is assumed to be of the general form used by Tongue and Yang [11]. This model consists of a spring and damper connected in parallel between the point of contact in a collision and the body of the vehicle. A schematic of the bumper is shown in Figure 2. The second assumption is that the bumper has a dead length, which is chosen to be 15 cm. This means that after a deflection of 15 cm, the bumper will have deflected its entire length, and ceases to be an active component in the dynamics of the system.

5.1.1 Initial Model

The components of the bumper model were initially assumed to be linear in nature. In a low ΔV collision the bumper should protect the body from deformation. If there is no deformation in the body, then any system dynamics due to the body structure will not be active. Thus these dynamics can be ignored, and the CDM simplifies to a bumper model and a lumped mass. The schematic of this system is shown in Figure 2, and its equations of motion are given as (1).

$$m\ddot{x}_b + c_b\dot{x}_b + k_bx_b = 0 \tag{1}$$

We shall assume critical damping in the following derivations. The quantities ω_n, ζ, and c_{crit} are defined as usual.

$$\omega_n := \sqrt{\frac{k_b}{m}} \tag{2}$$

$$\zeta = \frac{c_b}{c_{crit}} \tag{3}$$

$$c_{crit} := 2\sqrt{k_b m} \tag{4}$$

The assumption of critical damping implies that ζ is unity. Thus, the solution to (1) is:

$$x_b(t) = [x_b(0) + [\dot{x}_b(0) + \omega_n x_b(0)]\, t]\, e^{-\omega_n t} \tag{5}$$

Taking the derivative of (5) with respect to time and setting it equal to zero, the time at which maximum deflection occurs (t_{max}) is found to be

$$t_{max} = \frac{\dot{x}_b(0)}{\omega_n\, [\dot{x}_b(0) + \omega_n x_b(0)]} \tag{6}$$

Given the boundary conditions

$$\begin{aligned} x_b(0) &:= 0 \\ \dot{x}_b(0) &:= 2.2353\,\mathrm{m/sec} = 5\ \mathrm{MPH} \\ x_b(t_{max}) &:= 15\,\mathrm{cm} \end{aligned}$$

and substituting (6) into (5) (using $t_{max} = \frac{1}{\omega_n}$) yields the following expression for ω_n:

$$\omega_n = \frac{\dot{x}_b(0) e^{-1}}{x_b(t_{max})} \tag{7}$$

Applying the above derivation to identify parameters for a test vehicle with a mass of 2000 kg, the following values were obtained.

$$\begin{aligned} k_b &= 6.01\mathrm{x}10^4 \quad (\mathrm{N/m}) \\ c_b &= 2.19\mathrm{x}10^4 \quad (\mathrm{Nsec/m}) \end{aligned}$$

The response of the mass-bumper system colliding with a wall at 5 MPH is shown in Figure **3.** Increasing the velocity to the standard NHTSA impact velocity of 35 MPH yields the response shown in Figure 4. Figure 5 shows the forces generated by the bumper in the collision. This plot reveals the problem with a model relying on linear elements. As the vehicle initial velocity is increased, the force generated by the damper element in the bumper increases linearly with a slope of c_b. If the vehicle mass is held constant, increasing the initial velocity will eventually cause there to be zero deflection in the bumper during the collision. This is a phenomenon known as damper lockout. Because high velocity collisions should cause deflection of the bumper to its dead length, the exhibited behavior is undesirable. The following section details a model modification to alleviate this problem.

5.1.2 Final Model

In order to eliminate the damper lockout behavior observed at high velocity, the bumper's damping characteristics require modification. Equation (8) shows a non-linear damper relationship that has been used to address the problem.

$$F_{damper} := \frac{C_b x_b^2}{\sqrt{\gamma + \dot{x}_b^2}} \tag{8}$$

A plot of this force-velocity relationship for several values of γ is shown in Figure 6. Note that γ determines the slope of the function as it passes through the origin. As the velocity diverges from zero, the function asymptotes to a maximum value of C_b. Since the maximal force level is bounded, the damper cannot transmit an unbounded level of force and thus the problem of damper lockout is avoided.

Substituting (8) into (1) gives the new equation of motion for the bumper system.

$$m\ddot{x}_b + \frac{C_b \dot{x}_b}{\sqrt{\gamma + \dot{x}_b^2}} + k_b x_b = 0 \tag{9}$$

The bumper system is now governed by a second order non-linear differential equation. There exist, no closed form analytical solutions to (9), and identification of the system parameters must be performed numerically. Section 6 presents a procedure to determine the system parameters via a numerical optimization.

5.2 Body Model

The model for the vehicle deformation dynamics is based on the observation by Emori [14] that vehicles in a head-on collision deform in a spring-like manner during the deformation phase and then stop, i.e. experience no significant recovery. Qualitatively, this spring absorbs energy but never releases it. Although appealing for its simplicity, this approach is not entirely correct. After the plastic deformation of the steel structure that occurs during the dynamic crush of the vehicle, there will be some elastic crush recovery. For the purposes of our collision dynamics model, this behavior will be ignored. This assumption is made based on the facts that such recovery is small compared to the residual crush, and that the NHTSA database does not supply the necessary information to include elastic recovery, thus precluding a validation of the model.

Campbell [15] observed that vehicles obey the relationship between residual crush and impact force shown in Figure 7. This is a linear relationship with a non-zero dependent variable intercept. The relationship can be expressed as (10). The coefficients k_c and F_o have the following physical interpretation: k_c is spring constant of the the energy absorbing spring, while F_o represents the force required to initiate deformation of the vehicle body.

$$\frac{F_{body}}{w} = k_c x_c + F_o \tag{10}$$

This relationship can be simplified because the collision dynamics model is one-dimensional, allowing the assumption that the force is distributed equally over the width of the vehicle. Thus, the w term can be moved to the right hand side of (10), and subsumed into the k_c and F_o terms. The resulting relation is (11).

$$F_{body} = k_c x_c + F_o \tag{11}$$

The energy absorbed by the vehicle body, E_{body}, in a collision is the integral of (11) with respect to x_c.

$$E_{body} = \int_0^{x_c} (k_c x + F_o)\, dx \tag{12}$$

In the NHTSA data, E_{body} is known for only one (E_{body}, x_c) pair. In order to identify parameters k_c and F_o, additional information is required. The parameter F_o has the physical interpretation of the force at which body deformation occurs, and its value may be determined if assumptions are made about the interaction between the bumper and the vehicle body.

The bumper is assumed to comply with the U.S. 5 MPH standard. Thus the break out force of the body should be equal to the maximum force generated in the bumper in a 5 MPH collision. Once F_o is found, the (E_{body}, x_c) pair can be used to determine k_c. The procedure for the determination of this parameter is detailed in Section 6.

The body model given above is a very simple approach to describing the behavior of a very complex system. This simple model does provide a reasonable estimate of collision behavior, and its parameters are completely described by the NHTSA data. More complex models could be developed, but it would not be possible to determine additional model parameters due to the limited nature of the NHTSA database.

5.3 Completed Model

With the bumper and body subsystems defined, the entire vehicle collision dynamics model can be pieced together. The individual model used to derive Equations (13-15) is shown in Figure 8 while a complete car model (front and rear) is shown in Figure 9. The behavior of the CDM can be broken into three distinct phases having different associated state equations. Phase 1 is the bumper only phase, with state equations given by (13). In this phase, the bumper force has not reached the critical value F_o, and thus the body dynamics are not active. Once the the force in the bumper has reached F_o, the CDM enters phase two, given by (14). This phase contains the entire dynamics of the system. The system remains in this phase until the collision ends, or the bumper crushes to its dead length. If the bumper is crushed to its dead length, the CDM enters phase three, governed by (15). In this phase the bumper dynamics are no longer present, leaving only the body dynamics.

- Phase I: Bumper system active

$$\begin{aligned} \dot{x_1} &= -\tfrac{k_b(x_2-L_b)}{m} + \tfrac{C_b x_1}{(m+m_o)\sqrt{\gamma+x_1^2}} \\ \dot{x_2} &= x_1 \\ \dot{x_3} &= \dot{x_1} \\ \dot{x_4} &= \dot{x_2} \end{aligned} \tag{13}$$

- Phase 11: Bumper and body systems active

$$\begin{aligned} \dot{x_1} &= \tfrac{k_c(x_4-x_2-L_c-\delta)}{m} - \tfrac{k_b(x_2-L_b)}{m} + \tfrac{C_b x_1}{m\sqrt{\gamma+x_1^2}} \\ \dot{x_2} &= x_1 \\ \dot{x_3} &= -\tfrac{k_c(x_4-x_2-L_c-\delta)}{m} \\ \dot{x_4} &= x_3 \end{aligned} \tag{14}$$

- Phase 111: Body system active

$$\begin{aligned} \dot{x_1} &= 0 \\ \dot{x_2} &= 0 \\ \dot{x_3} &= -\tfrac{k_c(x_4-x_2-L_c-\delta)}{m} \\ \dot{x_4} &= x_3 \end{aligned} \tag{15}$$

To implement the CDM with (13), (14), and (15), additional logic is required. This logic is necessary to determine when to switch phases, to keep the body spring from elastically recovering, and to detect the beginning and end of the collision. Section 8 addresses these issues.

6 CDM Parameter Identification

The identification of the unknown model parameters is a multi-step process. First, the bumper parameters k_b, C_b, and y must be found via a non-linear optimization. After these values have been determined, a 5 MPH collision simulation is performed to calculate the maximum force generated in the bumper. Using this data, it is then possible to analytically determine the body spring constant, k_c, and the associated preload distance, δ.

6.1 Bumper Parameter Identification

The non-linear bumper model requires the identification of three parameters. The method used t,o do this is as follows:

1. Specify the slope of the damper force-velocity function at the origin.
2. Find C_b in terms of k_b.
3. Find γ in terms of k_b.
4. Find the k_b value that allows a 5 MPH bumper system simulation to match boundary conditions.
5. Solve for the associated C_b and y values.

The first step in the bumper parameter identification is to specify the behavior of the bumper about the system state-space origin. To do this, the Jacobian linearization of the system is found. The linearization of (9) about $(x_{1,0}, x_{2,0})$ is

$$\begin{bmatrix} \delta \dot{x_1} \\ \delta \dot{x_2} \end{bmatrix} = \begin{bmatrix} \frac{-C_b}{m\sqrt{\gamma + x_{1,0}^2}} + \frac{C_b x_1^2}{m(\gamma + x_{1,0}^2)^{\frac{3}{2}}} & -\frac{k_b}{m} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix} \tag{16}$$

Upon evaluation about the operating point of $(x_{1,0}, x_{2,0}) = (0,0)$, (16) simplifies to

$$\begin{bmatrix} \delta \dot{x_1} \\ \delta \dot{x_2} \end{bmatrix} = \begin{bmatrix} \frac{-C_b}{m\sqrt{\gamma}} & -\frac{k_b}{m} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix} \tag{17}$$

The damping constant in the linearized system is the (1,1) entry of the system matrix in (17). The damping ratio is defined as $\zeta := \frac{c}{c_{crit}}$. An expression for γ in terms of the other unknown bumper parameters, ζ, and the vehicle mass is

$$\gamma = \frac{C_b^2}{4\zeta^2 k_b m} \tag{18}$$

Taking the maximum force generated in the damper to be one half of that generated in the spring,

$$C_b = \frac{1}{2} k_b x_{max} \tag{19}$$

allows the simplification of (18) to (20).

$$\gamma = \frac{k_b x_{max}^2}{16\zeta^2 m} \tag{20}$$

Now that all unknown bumper parameters have been expressed as a function of k_b, a one-dimensional gradient descent optimization can be performed. This optimization attempts to drive the maximum deflection in a 5 MPH collision simulation to $x_{max} = 15$ cm by varying k_b. In practice it has been found that the algorithm is generally able to achieve errors of less than one millimeter in the deflection.

6.2 Body Parameter Identification

As stated in Section 5.2, once the bumper parameters are known it is possible to determine the body parameters from the NHTSA data. Equation (10) relates the force generated in the body to the residual crush in the body, X,, and a breakout force, F,. For the CDM, this breakout force will be treated as a preload on the body spring. Thus, to complete the model, the parameters k_c and δ must be determined. Modifying (10) to account for the preload, (21) is obtained.

$$E_{body} = \int_{\delta}^{x_c} k_c x dx \tag{21}$$

The maximum force generated in the bumper in a 5 MPH collision is defined to be $F_{b,max}$. Equating this force to the preload force in the body spring, then solving for the preload distance, δ, gives

$$\delta = \frac{F_{b,max}}{k_c} \tag{22}$$

By carrying out the integration in (21) and substituting for 6, a relationship with C, as the only unknown can be found.

$$\frac{1}{2} x_c^2 k_c^2 - E_{body} k_c - \frac{F_{b,max}^2}{2} = 0 \tag{23}$$

Taking the positive root of this polynomial gives the value for k_c, which can then be substituted into (22) to find the value of 6.

7 Two Car CDM

The CDM developed in Section 5.3 is a single car model for simulation of collisions with a fixed object. This model was necessary for the identification of system parameters from the NHTSA data. For the CDM to be useful in the PSP it must be modified to accurately model the interaction between two vehicles in a collision. This modification consists of taking two systems of the form shown in Figure 9 and connecting them in series. Unfortunately, realization of this connection in software is not simple as the interface between the two systems is not in a form well suited for a state-space equation description. To model the interaction as three sets of equations for each car as in Section 5.3, system augmentation is required. Because system augmentation has some undesirable numerical properties, another approach is used. A multiple model implementation is developed that treats the bumper damper non-linearity as a piece-wise linear function by subdividing the collision phases which involve the bumper.

7.1 Augmented Model Implementation

In order to describe the interaction between two vehicle bumpers as the three sets of equations as in Section 5.3, we will add an intermediate mass to the system between the vehicle bumpers. The same type of augmentation is also necessary to interface between the bumper and body spring. These masses are added so that the position and velocity of each bumper and body spring end is represented by a state variable, and is thus known at each simulation time step. If the masses are kept small relative to the lumped vehicle mass, their effect of the simulation results will be negligible. Figure 13 is a schematic of the two car CDM with the intermediate masses added.

A simulation of a head-on collision between two 1986 Ford Taurus passenger cars is shown in Figures 10 and 11. The results of the simulation are as expected. The same forces are applied to each vehicle, causing the same deformation. The final velocity is as predicted by a conservation of momentum calculation. While simulation results are accurate, the simulation time for the augmented model is approximately ten minutes on a DEC 5000 workstation. The length of simulation is due to the size of integration steps required to obtain accurate results. The inclusion of the small masses in the augmentation process introduced a natural frequency on the order of $\omega_n = 1\text{x}10^6$ between an augmentation mass and the body spring. It is common to set the integration time step, Δt_{int}, to be

$$\Delta t_{int} = \frac{1}{10\omega_n} \tag{24}$$

Thus the integration time steps must be extremely small. Because of the excessive time required to perform a simulation using the two car augmented CDM, it has been rejected for use in the PSP.

7.2 Multiple Model Implementation

The multiple model implementation of the two car CDM approaches the system interface problem in a different manner than discussed in the previous section. Much like the technique presented in Section 5.3, this implementation breaks the collision into phases which can each be treated with a single model. For each vehicle there are the three phases presented in Section 5.3. If each phase for each vehicle can potentially interact with each phase from the other vehicle, nine equation sets are required to completely describe a collision. Additionally, a simplification regarding the non-linear bumper is required for the tractability of the equation sets involving the bumper. This simplification is the conversion of the non-linear relationship between velocity and force in the bumper damper to a piece-wise linear one. An approximation of the non-linear function is shown in Figure 12. While this approximation simplifies the equations, it complicates the model by adding six equation sets.

7.2.1 Equation Sets

Deriving and providing proper switching logic for 15 equation sets is a daunting task. However, the physics of the problem are such that not all of the phase combinations are physically possible. Also, simulation results have shown that several other combinations have negligible affect on the simulation. By removing these combinations from the list of possibilities, the final model is greatly simplified. The final multiple model implementation is based on the schematic in Figure 14. The model inputs are the position and velocity of the vehicle masses. Outputs to the PSP are the dynamic forces acting on those masses due to the collision. This model consists of three states equations and two output equations for each phase combination as follows:

- Bumpers dynamics active.

– Both bumper dampers in linear region.

$$\begin{aligned}
\dot{x_1} &= v_f \\
\dot{x_2} &= \frac{k_{b,f}(x_2-x_1-L_{b,f})-k_{b,r}(x_3-x_2-L_{b,r})-c_{b,f}v_f-c_{b,r}v_r}{-(c_{b,f}+c_{b,r})} \\
\dot{x_3} &= v_r \\
F_f &= k_{b,f}(x_2-x_1-L_{b,f})+c_{b,f}(\dot{x_2}-\dot{x_1}) \\
F_r &= -k_{b,r}(x_3-x_2-L_{b,r})-c_{b,r}(\dot{x_3}-\dot{x_2})
\end{aligned} \tag{25}$$

– Front vehicle bumper in linear region, rear vehicle bumper in constant region.

$$\begin{aligned}
\dot{x_1} &= v_f \\
\dot{x_2} &= \frac{k_{b,f}(x_2-x_1-L_{b,f})-k_{b,r}(x_3-x_2-L_{b,r})-c_{b,f}v_f-C_{b,r}}{-c_{b,f}} \\
\dot{x_3} &= v_r \\
F_f &= k_{b,f}(x_2-x_1-L_{b,f})+c_{b,f}(\dot{x_2}-\dot{x_1}) \\
F_r &= -k_{b,r}(x_3-x_2-L_{b,r})+C_{b,r}
\end{aligned} \tag{26}$$

– Front vehicle bumper in constant region, rear vehicle bumper in linear region.

$$\begin{aligned}
\dot{x_1} &= v_f \\
\dot{x_2} &= \frac{k_{b,f}(x_2-x_1-L_{b,f})-k_{b,r}(x_3-x_2-L_{b,r})-C_{b,f}-c_{b,r}v_r}{-c_{b,r}} \\
\dot{x_3} &= v_r \\
F_f &= k_{b,f}(x_2-x_1-L_{b,f})-C_{b,f} \\
F_r &= -k_{b,r}(x_3-x_2-L_{b,r})-c_{b,r}(\dot{x_3}-\dot{x_2})
\end{aligned} \tag{27}$$

– Both vehicle bumpers in constant region.

$$\begin{aligned}
\dot{x_1} &= v_f \\
\dot{x_2} &= \frac{k_{b,f}v_f+k_{b,r}v_r}{k_{b,f}+k_{b,r}} \\
\dot{x_3} &= v_r \\
F_f &= k_{b,f}(x_2-x_1-L_{b,f})-C_{b,f} \\
F_r &= -k_{b,r}(x_3-x_2-L_{b,r})+C_{b,r}
\end{aligned} \tag{28}$$

- Body dynamics active.

$$\begin{array}{rcl} \dot{x_1} & = & \dot{x_2} \\ \dot{x_2} & = & \frac{k_{c,f}v_f + k_{c,r}v_r}{k_{c,f}+k_{c,r}} \\ \dot{x_3} & = & \dot{x_2} \\ F_f & = & k_{c,f}(x_1 - p_f - L_{c,f} - \delta_f) \\ F_r & = & -k_{c,r}(p_r - x_3 - L_{c,r} - \delta_r) \end{array} \quad (29)$$

7.2.2 Switching Logic

In order to correctly implement (25) through (29) the proper logic for model switching must be in place. The logic steps for equation set determination are as follows:

1. Check to see if the vehicles are already in the body dynamics active phase. If so, use (29).

2. If vehicles are in the bumper only phase, assume that both bumpers are in the linear region. Calculate the response via (25).

3. Find the forces in both bumper dampers, $F_{b,f}$ and $F_{b,r}$, from the response calculated in the previous step.

4. Check these forces against the maximum forces possible in each bumper, $C_{b,f}$ and $C_{b,r}$.

5. If $F_{b,f} < C_{b,f}$ and $F_{b,r} < C_{b,r}$, use the previously calculated response.

6. If $F_{b,f} < C_{b,f}$ and $F_{b,r} > C_{b,r}$, use (26).

7. If $F_{b,f} > C_{b,f}$ and $F_{b,r} < C_{b,r}$, use (27).

8. If $F_{b,f} > C_{b,f}$ and $F_{b,r} > C_{b,r}$, use (28).

9. Find the deflection in each bumper. If either bumper has crushed to its dead length, the model should set a register signifying that it is in the body dynamics active phase for the rest of the collision.

While the multiple model implementation of the two car CDM is not as elegant as the augmented model version, its simulation time is approximately four and a half times faster. When coded in a C function for use with Matlab, the multiple model CDM is 150 times as fast as the augmented model version, bring the simulation time down to approxirnately four seconds. This boost in performance makes the multiple model CDM the best choice for implementation in the PSP.

8 Implementation Issues

The two car CDM developed in Section 7.2 fulfills all requirements for implementation in the PSP. However, it does require additional support to be of use. As developed in Section 7.2 the CDM consists of several sets of equations and the logic required to select which equation set to use. This model performs comparably to the augmented model, with the advantage of greatly decreased simulation time. However, the model lacks the intelligence to sense when a collision has started and finished. The model requires additional support from the integration code as well. CDM events occur on a time scale much smaller than events of the platoon during nominal operation. This fact requires special attention from the numerical integration algorithm used in the PSP.

8.1 Collision Detection

The CDM as presented in Section 7.2 assumes that the cars are alway in contact,. This assumption leads to problems if not explicitly dealt with by collision detection logic. The collision detection module is used to decide when the vehicles have come into contact, and when the collision contact has ended. This information is passed from the detection module to the CDM so that appropriate action can be taken. When no contact is present, the CDM force outputs, F_f and F_r, are set to zero, as are the derivatives of the CDM states.

The detection of collision initiation in a one-dimensional model is a trivial matter. The algorithm takes the position of the vehicle lumped masses, adds the appropriate body and bumper lengths, then compares the two values. If the front bumper of the rear vehicle passes the rear bumper of the front vehicle, a collision has occurred. Once a collision is detected the algorithm signals the CDM. Upon receiving this signal, the CDM sets its initial conditions to match the current configuration of the vehicles. Numerical integration then continues until the collision has ended. During the collision, the detection module is deactivated so as to not repeatedly signal the initiation of a collision.

Collision cessation is not as simple to detect as initiation. This event is actually detected internal to the CDM code. Two criteria are used to determine the end of a collision, dependent on the severity of the collision. If the collision involved only the vehicle bumpers, then it is deemed completed when the bumpers separate. If the collision involved permanent deformation of the car bodies, it is deemed completed when either body spring attempts to elastically recover. After this the CDM signals the collision detection module to reactivate and begin checking for collisions again.

8.2 Numerical Integration Algorithm

In the simulation of a platoon under nominal operating conditions the integration time step is on the order of milliseconds. The characteristic response times of all systems involved for nominal operation are large enough that accurate results can be obtained with such a large step size. This allows simulations of several minutes of simulated time to be performed in a reasonable amount of real time. The occurrence of a collision drastically changes this. Simulations have shown that collisions occur in under ten milliseconds. Using the integration step size that was used during nominal operation will lead to erroneous results.

8.2.1 Adaptive Algorithms

The traditional approach to handling non-linear dynamic system numerical simulation is to use integration algorithms that offer dynamic step size adaptation. The use of these algorithms in their standard forms as the integration algorithm for the PSP has been rejected for two reasons. First, their adaptation is non-optimal for this system. Second, their performance upon initiation of contact is very poor.

The adaptation algorithms have been developed for systems where the characteristic times are unknown or only known within wide bounds. The PSP with the CDM has characteristic times known to be within two bound sets that are vastly different. The traditional adaptation algorithms will attempt to find integration time steps that fall between these sets before finally settling within one them. If the integration algorithm can be forced to operate within the correct bound set upon initiation or completion of a collision, many unnecessary calculations can be avoided.

The initiation of a collision has also proved to be problematic for the performance of the adaptive integrators. If the platoon simulation is operating in a nominal mode, the integration step size may be as great as ten milliseconds. This may cause the simulation to step entirely past a collision, or detect it only once it is half way completed. The results in this type of situation are poor.

8.2.2 PSP Algorithm

The solution to the problems discussed above requires the modification of one of the adaptive integration algorithms so that the knowledge of the system can be used to increase performance. If the integration algorithm can detect the onset of a collision, then switch the step size ranges accordingly, a boost in simulation performance can be obtained.

The PSP integration algorithm is a modification of the Runge-Kutta high order method. Essentially, the algorithm has been changed so that it receives information from the CDM about whether or not a collision is occurring. With this information the algorithm can set the minimum and maximum integration step sizes to the proper values. The CDM carries out this communication via augmentation with an additional state that serves as a flag. The flag state is identified to the PSP integration algorithm by having an initial condition value of infinity. Once all flag states have been identified, they can be checked by the integrator at each time step.

For the CDM to signal the integration algorithm about the presence or absence of a collision, it must set the flag state derivative value. The flag state has a derivative of zero if the platoon is operating under nominal conditions. If a collision occurs, the flag state derivative is set to one. This signals the integration algorithm to use the smaller step size range. If the flag state derivative transitions from zero to one at any of the subintervals used by the Runge-Kutta algorithm, that step is immediately started over with the small step size range. The integration code uses the small step size range until the simulation time passes the simulation time at which the collision was initially detected. This check is required to keep the integrator from toggling between ranges. After the initial collision time has passed, the integrator begins checking the flag state for the one to zero transition that, signals the end of a collision. Note that the integrator must check the status of all flag states so that it exhibits the proper behavior in the presence of multiple collisions.

9 Results and Conclusions

The end result of the development, work presented in this paper is a Simulink Collision Dynamics Model block for use in the PSP. This block is shown in a two car platoon Simulink block diagram in Figure 15. The block takes the vehicle states and identification information as inputs and returns the forces generated on each vehicle as the outputs. The internal topology of the CDM block is shown in Figure 16. This CDM block includes both the collision detection and collision dynamics algorithms, and it provides the communications between them. Both of these algorithms have been coded as compilable C functions for use with Matlab (CMEX Functions).

The performance of the CDM block when used with the PSP integration algorithm discussed in Section 8.2.2 is shown in Figures 17 through 19. These plots show the results of a head-on collision between two 1986 Toyota Celica passenger cars. The front, vehicle was initially at rest, while the rear vehicle had an initial velocity of 20 m/sec. The initial distance between the vehicle bumpers was 0.55 meters. During the simulation, no forces other than the collision forces were applied to the vehicles.

Figure 17 shows the velocity trajectories of the vehicle masses for both cars. During the first 27.5 milliseconds the vehicles remain at their initial velocities. After this time, the rear car has covered the initial separation distance and comes into contact with the front vehicle. At that time the vehicle begins deforming. This deformation generates forces on both vehicles, causing their velocities to change. At approximately 80 milliseconds the vehicles are both moving the same speed, as is expected. As there are no other forces acting on the vehicles in this simulation, they continue to move at the same velocity until the end of the simulation.

The motion of the vehicles' center of mass is shown in Figure 18. The front car is stationary at the beginning of the simulation, while the rear car is moving towards it. Upon initiation of contact, the front car begins moving while the rear car slows. At the end of the simulation both cars are moving such that the distance between them remains constant.

Figure 19 shows the performance of the rear vehicle bumper in the collision. The position time histories of both ends of the bumper are displayed in this plot. The bumper comes into contact with the front vehicle at the time shown. For approximately the next 25 milliseconds the bumper deforms. At the end of this time the bumper is completely deformed and body deformation is initiated.

After the complete deformation of the vehicle bumpers, body deformation begins. Figure 20 shows the lengths of the car bodies as a function of time. The distance from the center of mass to the end of the car body is 1.85 meters in each vehicle at the start of this simulation. Figure 20 shows that both vehicles crush equally, as is expected from two identical vehicles. Note that the crush in the vehicles remains at the maximum value and no elastic recovery occurs in the car bodies.

The collision dynamics model developed in this paper is simple to integrate into the Platoon Simulation Package. This CDM provides the forces generated on the masses of the cars during a collision as well as the deformations imparted on each vehicle. The addition of this module to the PSP will not greatly increase the time required for simulation of platoons involving collisions. The results of the test simulation discussed are as expected. It is therefore recommended that the model developed herein be used for the modeling of the physical interaction of vehicles in the Platoon Simulation Package.

References

[1] Hitchcock, A., "A Specification of an Automated Freeway with Vehicle-Borne Intelligence," *PATH Research Report UCB-ITS-PRR-92-18,* December 1992.

[2] Sheikholeslam, S., and Desoer, C.A., "Longitudinal Control of a Platoon of Vehicles; I: Linear Model," *PATH Research Report UCB-ITS-PRR-89-3,* August 1989.

[3] Sheikholeslam, S., and Desoer, C.A., "Longitudinal Control of a Platoon of Vehicles; 11: First and Second Order Time Derivatives of Distance Deviations," *PATH Research Report UCB-ITS-PRR-89-6,* December 1989.

[4] Sheikholeslam, S., and Desoer, C.A., "Longitudinal Control of a Platoon of Vehicles; 111: Nonlinear Model," *PATH Research Report UCB-ITS-PRR-90-1,* April 1990.

[5] Shladover, S.E., "Longitudinal Control of Automotive Vehicles in Close-Formation Platoons," *Journal of Dynamic Systems, Measurement, and Control,* Vol. 113, June 1991, pp. 231-241.

[6] Swaroop, D., "String Stability of Interconnected Systems: An Application to Platooning in Automated Highway Systems," *Ph. D. Dissertation,* Vehicle Dynamics Lab., U.C. Berkeley, December 1994.

[7] NHTSA Research and Development Vehicle Crash Test Database, Available from Barbara C. Hennessy, NHTSA, U.S. Department of Transportation, 400 Seventh Street, S.W., Washington, D.C. 20590.

[8] Strother, C.E, Woolley, R.L., and James, M.B., "A Comparison Between NHTSA Crash Test Data and CRASH3 Frontal Stiffness Coefficients," Society of Automotive Engineers Paper No. 900101.

[9] Reizes, H., *The Mechanics of Vehicle Collisions,* Charles C. Thomas, Springfield, Illinois, 1973.

[10] Tongue, B.H., and Moon, A., "Low Speed Collision Dynamics and Control: Year One Report," *PATH Research Report UCB-ITS-PRR-95-25,* August 1995.

[11] Tongue, B.H., Yang, Y.T., and White, M.T., "Platoon Collision Dynamics and Emergency Maneuvering I: Reduced Order Modeling of a Platoon for Dynamical Analysis," *PATH Research Report UCB-ITS-PRR-91-15,* August 1991.

[12] Tongue, B.H., and Yang, Y.T., "Platoon Collision Dynamics and Emergency Maneuvering 111: Platoon Collision Models and Simulations," *PATH Research Report UCB-ITS-PRR-94-02*, February 1994.

[13] Tongue, B.H., and Yang, Y.T., "Platoon Collision Dynamics and Emergency Maneuvering IV: Intra-Platoon Collision Behavior and A New Control Approach for Platoon Operation During Vehicle Exit/Entry - Final Report," *PATH Research Report UCB-ITS-PRR-94-25*, November 1994.

[14] Emori, R.,"Analytical Approach to Automobile Collisions", Society of Automotive Engineers, Paper No. 680016.

[15] Campbell, K.L., "Energy Basis for Collision Severity", Society of Automotive Engineers, Paper No. 740565.

[16] Strother, C.E, Woolley, R.L., James, M.B., and Warner, C.Y., "Crush Energy in Accident Reconstruction," Society of Automotive Engineers, Paper No. 860371.

[17] Karnopp, D.C., Margolis, D.L., and Rosenberg, R.C., *System Dynamics: A Unified Approach*, John Wiley and Sons, Inc., New York, New York, 1990.

[18] Greenwood, D.T., *Principles of Dynamics*, Prentice Hall, Englewood Cliffs, New Jersey, 1988.

[19] Meriam, J.L., and Kraige, L.G., *Engineering Mechanics: Dynamics*, John Wiley and Sons, Inc., New York, New York, 1986.

[20] Thomson, W.T., *Theory of Vibration with Applications*, Prentice Hall, Englewood Cliffs, New Jersey, 1988.

[21] *MATLAB Reference Guide*, The Mathworks, Natick, Massachusetts, August 1992.

[22] *SIMULINK User's Guide*, The Mathworks, Natick, Massachusetts, March 1992.

[23] Balas, Gary J., Doyle, John C., Glover, Keith, Packard, Andy, and Smith, Roy, *μ-Analysis and Synthesis Toolbox*, MUSYN Inc. and The Mathworks, Inc., January 1994.

[24] Kelley, A., and Pohl, I., *A Book on C: Programming in C*, Benjamin/Cummings Publishing Company, Inc., Redwood City, California, 1990.

[25] Wylie, R.C., and Barrett, L.C., *Advanced Engineering Mathematics*, McGraw-Hill, Inc., New York, New York, 1995.

[26] Lipschutz, S., *The Theory and Problems of Linear Algebra,* McGraw-Hill, Inc., New York, New York, 1991.

[27] Spiegel, M.R., *The Theory and Problems of Advanced Mathematics for Engineers and Scientists*, McGraw-Hill, Inc., New York, New York, 1992.

[28] Varaiya, P. and Shladover, S., Sketch of an IVHS Architecture," PATH Research Report, UCB-ITS-PRR-91-3, 1991.

A Figures

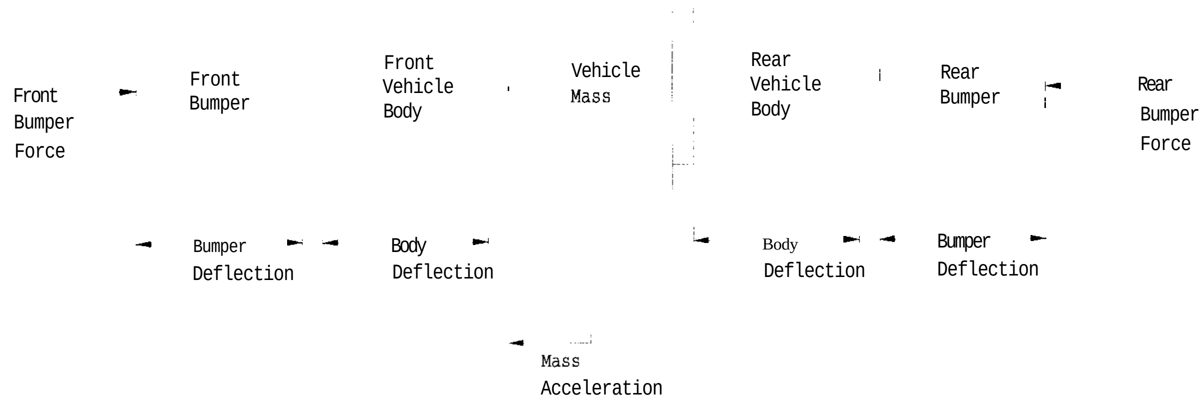

Figure 1: Schematic of Collision Dynamics Model.

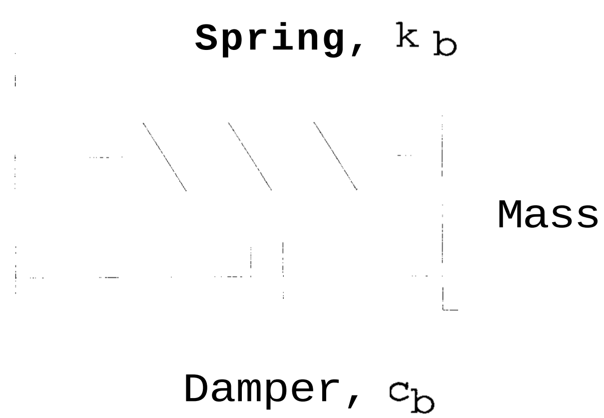

Figure 2: Schematic of bumper model.

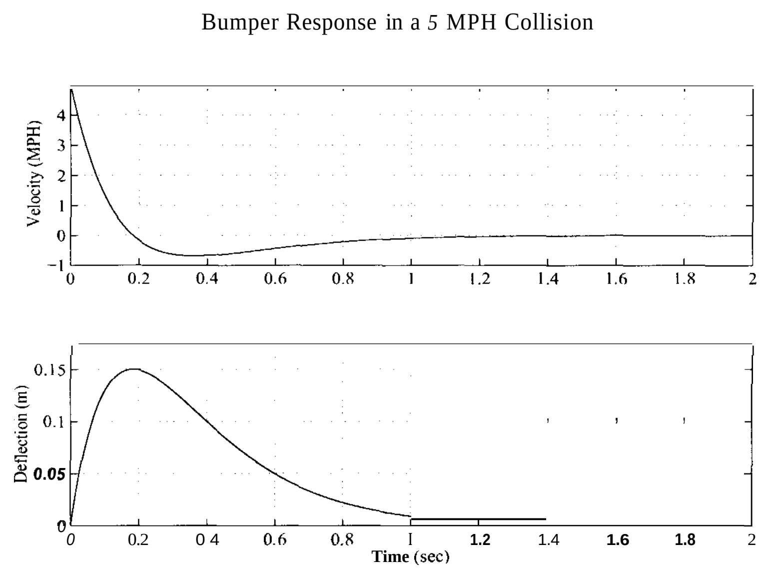

Figure 3: Deflection (x_b) and velocity (v_b) response of bumper system with a linear damper in a 5 MPH collision simulation.

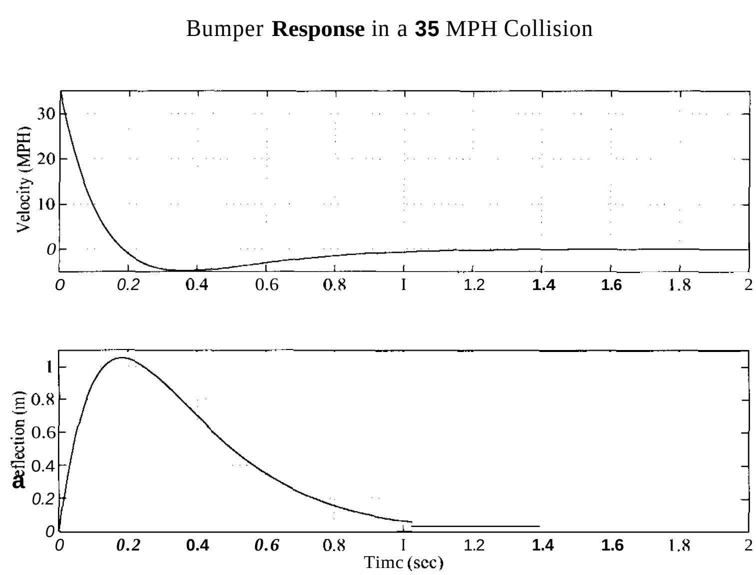

Figure 4: Deflection (x_b) and velocity (v_b) response of bumper system with a linear damper in a 35 MPH collision simulation.

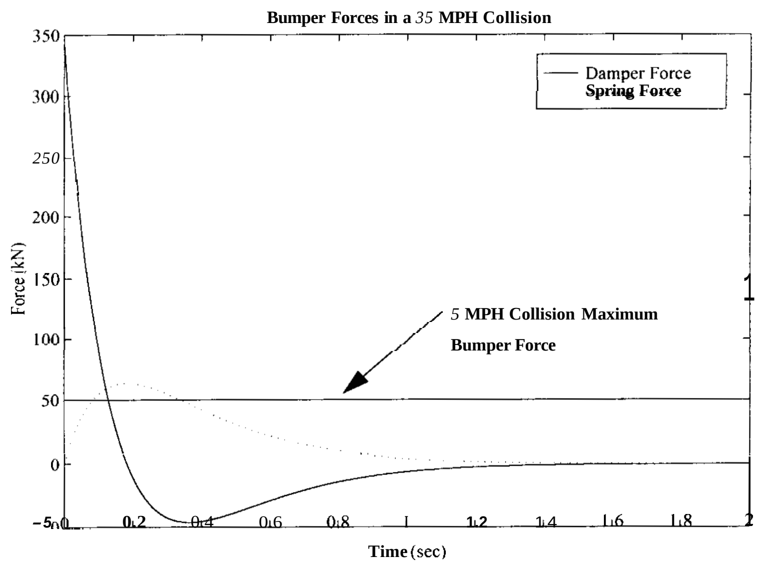

Figure 5: Forces in the bumper system with a linear damper in a 35 MPH collision simulation.

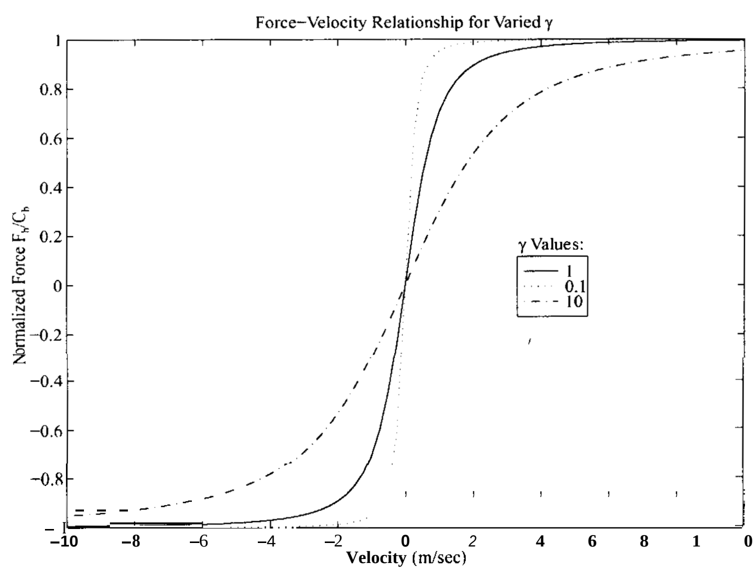

Figure 6: Non-linear damper force-velocity relationship normalized by the maximum damper force, C_b, for various y values. The parameter y determines the slope of the function as it passes through the origin.

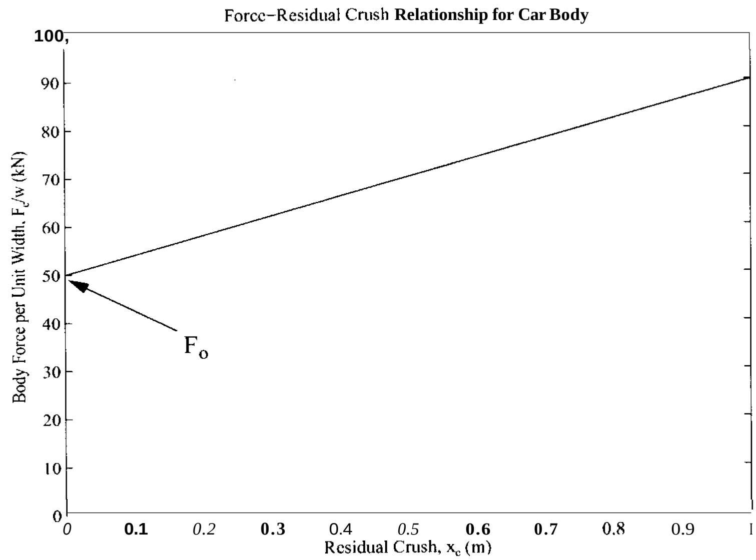

Figure 7: Force-residual crush relationship introduced by Campbell [15] for collisions of greater than 5 MPH.

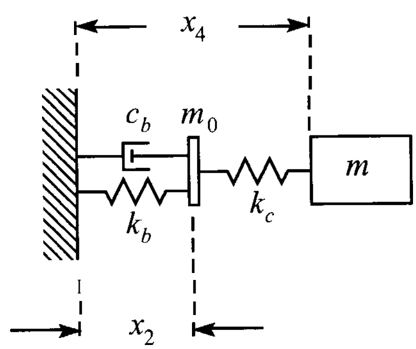

Figure 8: Front (or Rear) Bumper and Car Body Model

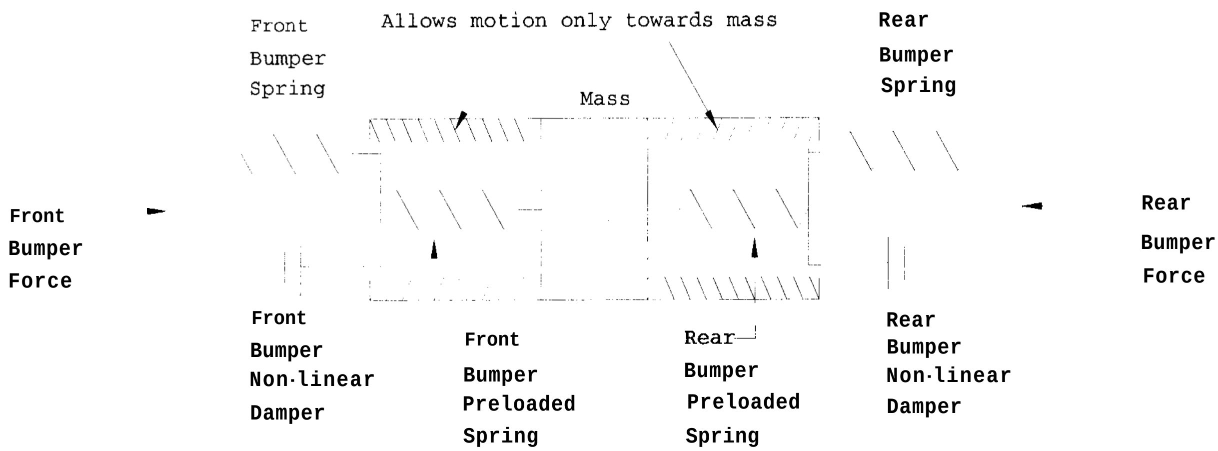

Figure 9: Schematic of the complete collision dynamics model with both front and rear bumper and body systems.

Augmented two car system collision simulation force response

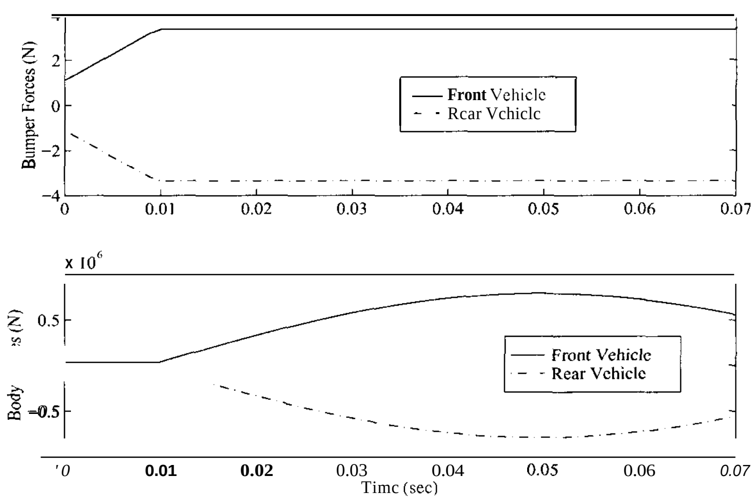

Figure 10: Collision forces generated by the bumpers and bodies of two Ford Taurus passenger cars in a head-on collision. Note that the body forces have a non-zero initial value due to the preload in the spring. Also note that the forces reach a maximum value, then recover due to the lack of logic limiting the elastic recovery in the body springs.

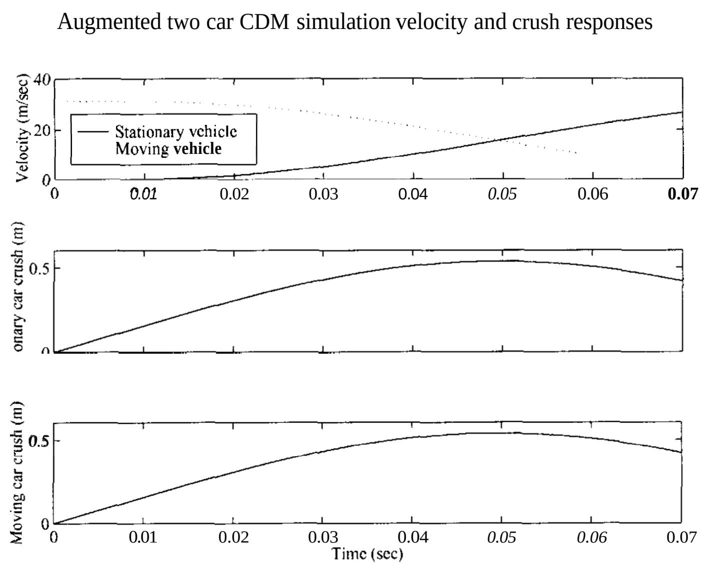

Figure 11: Bumper and body crushes, as well as the velocity response for a simulation of two 1986 Ford Taurus passenger cars in a head-on collision. Note that there is elastic recovery in the body springs due to the lack of control logic in the simulation. Also note that the velocity trajectories cross due to the lack of logic to detect the end of the collision.

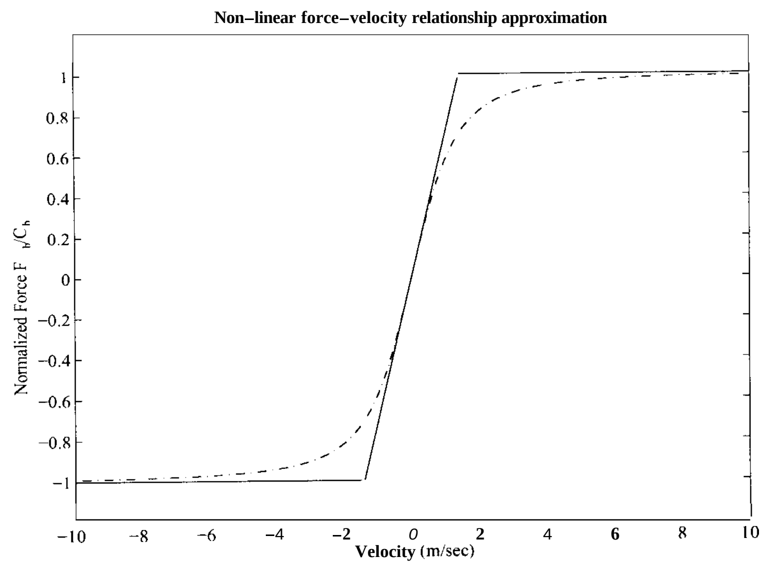

Figure 12: Approximation of the non-linear damping force introduced in Section 5.1.2 by a piecewise linear function.

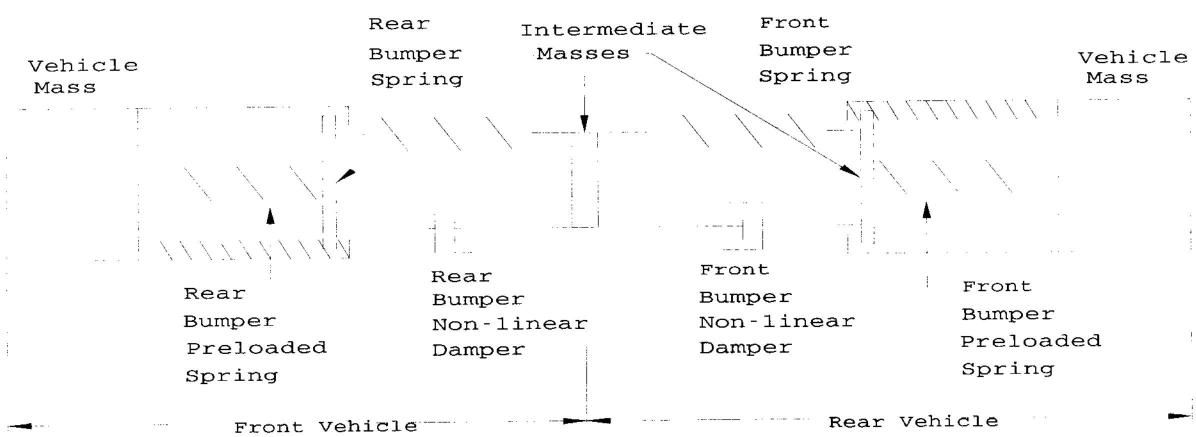

Figure 13: Two car CDM augmented to include intermediate masses at bumper-bumper and bumper-body junctions.

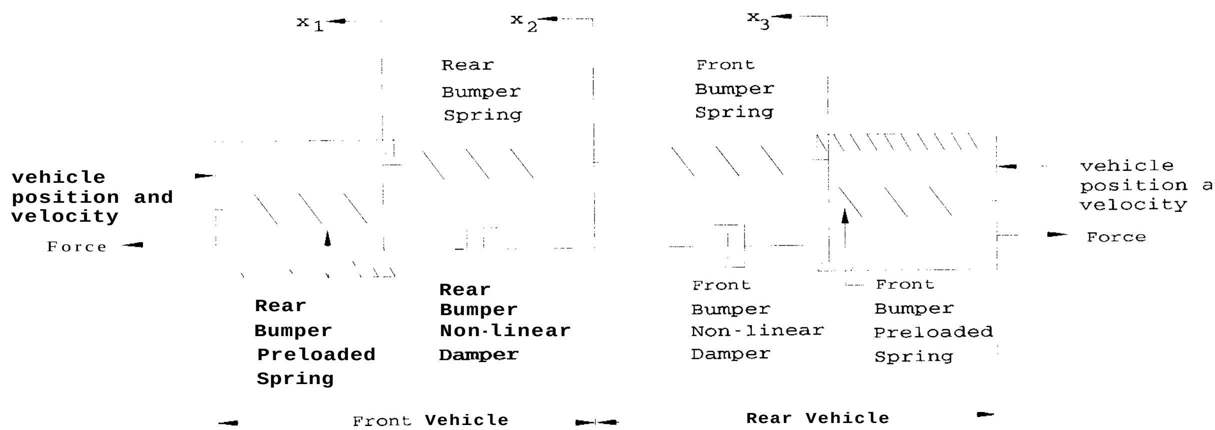

Figure 14: Two car CDM for the multiple model implementation described in Section 7.2.

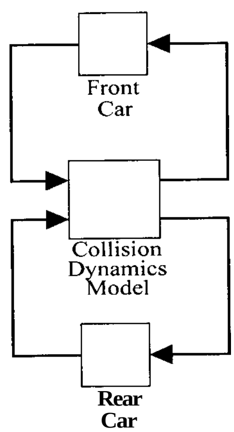

Figure 15: Simulink block diagram implementation in PSP. The Collision Dynamics Model block receives state and identification information from the vehicle block and returns the collision forces.

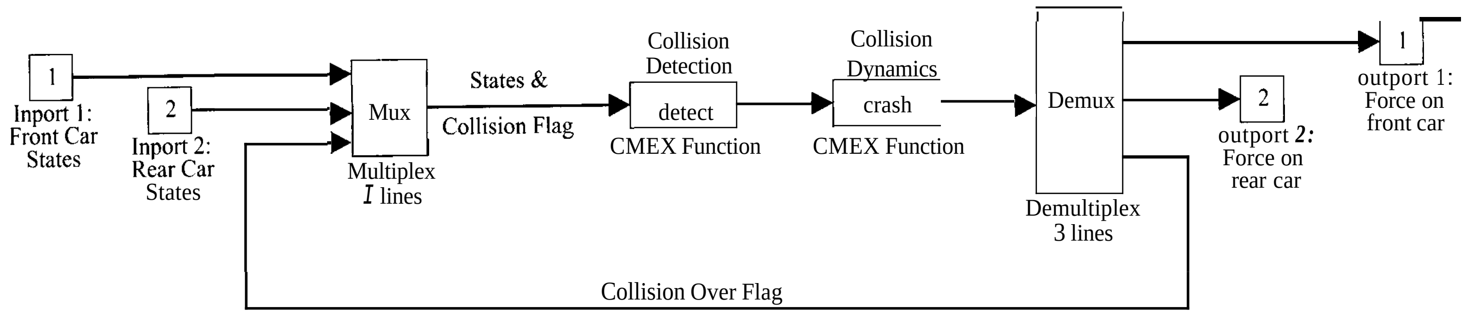

Figure 16: Simulink block diagram of the Collision Dynamics Model block. This block contains the collision detection and collision dynamics algorithms.

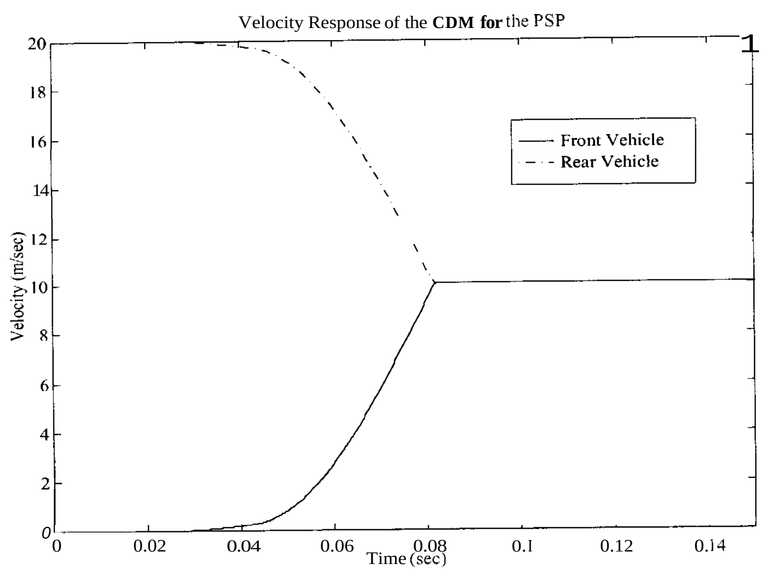

Figure 17: Velocity response of two 1986 Toyota Celica passenger cars in a head-on collision. The front car is stationary while the rear car is moving at 20 m/s initially.

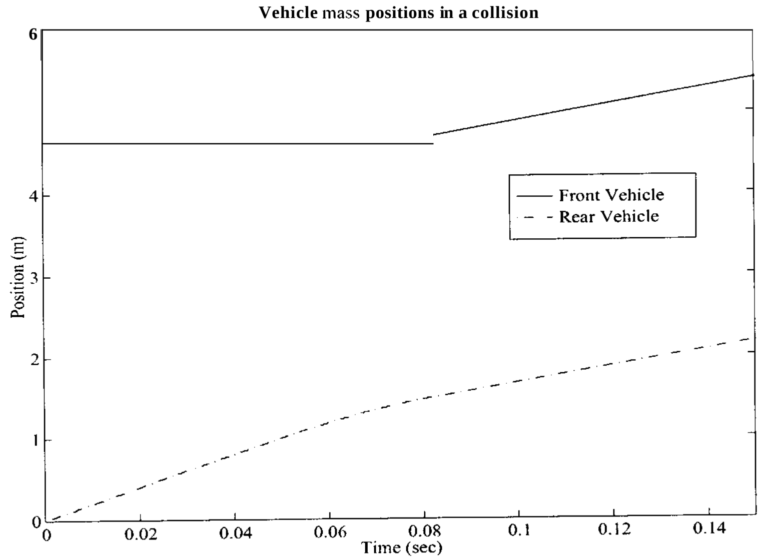

Figure 18: Position response of the center of mass of two 1986 Toyota Celica passenger cars in a head-on collision. The vehicles begin 0.55 meters apart.

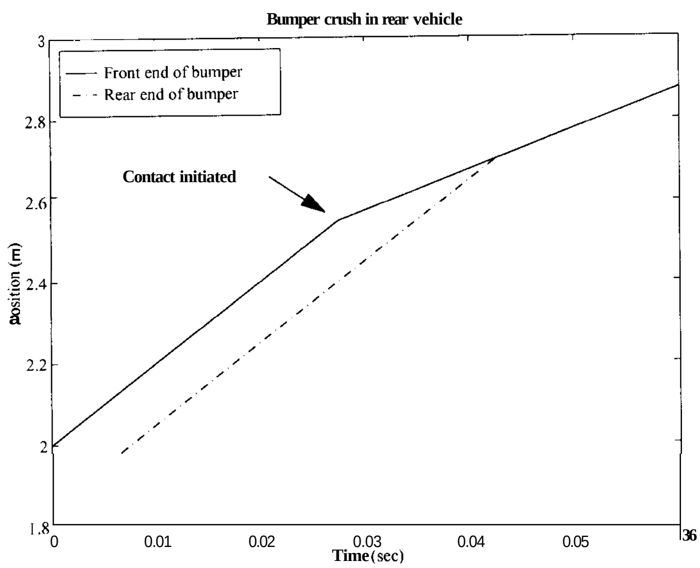

Figure 19: Bumper end point positions of rear Toyota Celica in a $\Delta V = 20$ (m/sec) collision with another Toyota Celica.

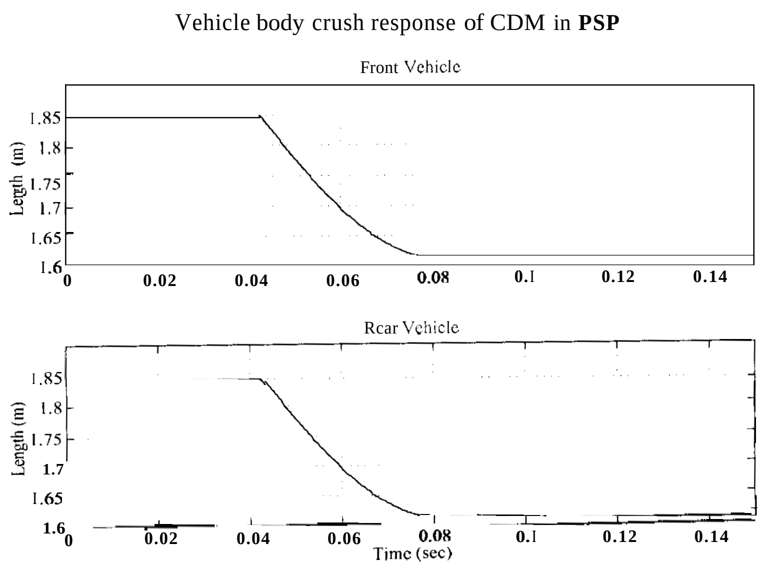

Figure 20: Body lengths of two 1986 Toyota Celica passenger vehicles involved in a $\Delta V = 20$ (m/s) head-on collision.