Predictability of Time-dependent Traffic Backups and Other Reproducible Traits in Experimental Highway Data

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PREDICTABILITY OF TIME-DEPENDENT TRAFFIC BACKUPS AND OTHER REPRODUCIBLE TRAITS IN EXPERIMENTAL HIGHWAY DATA

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Abstract

Traffic data from a 4-mile long congested rural road in Orinda, California, are used to show that traffic delays and vehicle accumulations between any two generic observers located inside a road section can be predicted from the traffic counts measured at the extremes of the section. The traffic model does not require “recalibration” on the day of the experiment, and works well despite what appears to be location-specific driver behavior.
1 INTRODUCTION

The purpose of this paper is to determine if traffic behavior upstream of a bottleneck can be predicted on a large scale, even in the presence of phenomena that are not reproducible at a more detailed level. The analysis focuses on traffic accumulations and waves, the main determinants of queue spill-backs (i.e., traffic backups). The paper develops a methodology to calibrate the parameters of a prediction procedure, and then demonstrates the calibration and prediction process with the data set described in Smilowitz et al. (1998). Familiarity with the data set in this reference is not necessary, but may help those readers who wish to extend or test the ideas about to be presented. As explained in that reference, the data set is available at: http://www.ce.berkeley.edu/~daganzo/spdr.html on the world-wide-web.

This data set contains detailed observations of both queued and unqueued vehicle arrivals upstream of an actuated traffic signal on two different days. These data were collected along a single lane of a two-lane highway with virtually no points of access or egress and negligible passing; see Figure 1. Smilowitz et al. (1998) describes the field experiment, the data reduction procedure that included (minimal) filtering of bad data, and the overall results. The data were summarized in the usual way by cumulative curves of vehicle number (N-curves). Visual inspection of the N-curves revealed that queues appeared to form and dissipate in predictable ways and showed no evidence of traffic instabilities at locations that were not queued. Interestingly, although the traffic signal pulses faded within one-half mile, other oscillations arose within the queue farther upstream and, in fact, grew in amplitude. These oscillations never propagated beyond the end of the queue, however.

Additional analysis of the N-curves has shown phenomena consistent with earlier findings, as well as other behavior that cannot be explained easily by current models. For example, the data showed that the forward propagation of disturbances first noted in Cassidy and Windover (1995) for unqueued traffic (i.e., when the travel times between observers include no appreciable delay) occurred repeatedly and consistently. This is interesting because the test site was four miles long and these disturbances had ample
time to disappear. It was apparent that an N-curve downstream of an unqueued (or “free flowing”) segment could be predicted quite accurately from the N-curve upstream of the segment simply by translating the upstream N-curve to the right by the free-flow travel time across the segment. As an illustration of this, Figure 2 shows the propagation of a disturbance from the upstream-most observer, N(1,t), to all the downstream observers.\(^1\) The curve N(x,t) represents the cumulative number of vehicles to pass observer \(x\) by time \(t\). Note that a similar headway was observed at locations 2, 3, 4, 5, and 6, and that there was no queuing delay anywhere between observers 1 and 6. When traffic was queued (i.e., travel times were larger than the minimum), as occurred between observers 6, 7 and 8, disturbances did not propagate in that manner and this is illustrated by the N-curves of these observers which do not include a gap \(G_1\). We shall see later that when traffic was queued its behavior was dictated, as one might expect, by downstream rather than upstream flows.

\(\text{Unexpected queued traffic behavior.}\) Looking at periods in which there were long queues (such as the one depicted in Figure 3), we see that the cusps of curve \(N(8,t)\), which corresponds to a location about 250 feet upstream of the traffic signal, were usually “rounded” and the troughs were usually “sharp”. This seems to indicate that the acceleration wave was transmitted from the location of the traffic signal to observer 8 sharply (as a “shock”) and that the deceleration toward the end of the queue was not.

The latter effect could be explained if most drivers were to decelerate in two stages: (i) first taking their foot off the acceleration pedal (coasting) as soon as they recognized that they would have to stop, and (ii) waiting to apply the brakes until the last minute. The signal to coast would then be transmitted very rapidly and the signal to brake more slowly. The passage of the coasting signal would be marked by the beginning of the curvature of each cusp, and the passage of the braking signal by the end of the curvature. If this were true, one would expect these curved cusps to grow from one observer to the next (e.g. from 8 to 7) and this appeared to happen in most cases.

\(^1\)Gaps similar to \(G_1\) were observed repeatedly, usually directly ahead of trucks and buses. Presumably, these heavy vehicles could not climb quickly a long uphill grade that led to the site of the Smilowitz et al.
However, the acceleration shocks did not always remain sharp.\textsuperscript{2} Close examination of the data reveals that both the cusps and the troughs could become rounded upstream of observer 7, and that the pattern was not consistent. We note, however, that the traffic signal could be seen by drivers from location 7 (1/4 mile away), but not from location 6 (1/2 mile away). This suggests that drivers may have been motivated to not miss the ‘green’ phase when they were close to the signal and that they might have driven differently farther upstream. The erratic pattern, however, suggests that wave propagation and the detailed shape of the N-curves were influenced by the idiosyncrasies of the particular drivers affected by each wave.\textsuperscript{3}

The particular form of smoothing that was observed is noteworthy because it is inconsistent with the kinematic wave (KW) theory of traffic flow. The observed driver behavior is also inconsistent with existing car-following theories, but this is not elaborated here because the focus of this paper is the prediction of accumulation with the kinematic wave theory.

If the KW theory with constant wave speed held (i.e., with a linear flow-density curve in the queued regime) as formulated in Newell (1993),\textsuperscript{4} we would expect the N-curves for observers 7, 6, 5, and 4 to be identical or very similar to N(8,t) except for a vertical and horizontal translation. There would be no “rounding” of the cusps and troughs. The KW theory also implies that if the flow-density relationship were curved rather than straight, then only the cusps or only the troughs should become smooth, depending on whether the flow-density relation were convex or concave. Furthermore, the particular form of smoothing should continue as one moves upstream from one observer to the next, until the oscillations disappear. Obviously, the long period oscillations that are shown in Figure 3 for curves N(4,t) and N(5,t) should not have arisen if the KW theory were valid at this fine level of detail.

\textsuperscript{2} An acceleration shock will remain sharp as long as drivers accelerate reasonably fast somewhat independently of the precise motion of the car in front.
\textsuperscript{3} Windover (1998) found erratic wave speeds but less smoothing when looking at freeway data. We conjecture that, with the threat of lane changing in a multilane traffic setting, drivers may be more motivated to follow closely and be less idiosyncratic.
\textsuperscript{4} Newell (1993) was first to relate the N-curves to the kinematic wave theory of traffic flow proposed by
In view of this, this paper seeks to determine if the KW theory will predict approximate vehicle accumulations and trip times between observers on a larger scale. The paper is organized as follows. First, a simple methodology that will be used to calibrate the KW theory and predict the N-curves is reviewed in section 2. This methodology is applied to experimental data in section 3. The results are interpreted in section 4. A brief summary of the main findings is then presented in section 5.

2 REVIEW OF KW THEORY WITH PIECEWISE LINEAR N-CURVES

The procedures in Newell (1993) can be streamlined when the N-curves are piecewise linear. This simplification will allow us to estimate the parameters of the prediction procedure in a simple way, and to obtain the predictions quickly and easily. This section reviews this methodology as it applies to a queued section enclosed by two observation points: “U”- upstream and “D”-downstream. The downstream N-curve will be labeled \( N_D \) and the upstream curve \( N_U \).

The use of piecewise linear approximations is justified because the KW procedure is a “contraction mapping” in the space of N-curves. That is, if the KW procedure is applied to two \( N_D \) curves then the maximum separation that results between the two predicted \( N_U \) curves is at most equal to the maximum separation between the two input \( N_D \) curves (Daganzo, 1997). Therefore, if one approximates to within an acceptable tolerance level a raw input \( N_D \) curve by a piecewise linear curve, and then applies the KW procedure to the piecewise linear curve, one can be assured that the result will be within the same tolerance level. This is useful because the KW procedure can be simplified

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5 Our study site is unique in that vehicles cannot pass for a long distance. If a single driver decides to slow down (s)he can affect the whole platoon of cars behind and have a large microscopic effect. This is less likely to happen in multi-lane freeway traffic. We are not sure whether this peculiarity of our site favors or disfavors the KW model, although we lean toward the latter position and believe that a successful test with the Smilowitz et al. (1998) data bodes well for similar tests with multi-lane freeway data.

6 A similar result holds for errors in the flow-density relation; i.e. the maximum vertical separation between two \( N_U \) curves built from two different flow-density relations cannot exceed the product of the maximum density discrepancy in the two relations and the length of the UD section.
significantly when dealing with piecewise linear N-curves. The simplifications arise from three postulates that are equivalent to the KW theory but pertain to operations on piecewise linear N-curves (Daganzo, 1997). For the purposes of the present paper, only the first two postulates in that reference will be needed. They are explained below.

The first postulate (stationary reproducibility) states that vehicle accumulations should be replicable within a queue for any (long) time interval in which the average flow for the interval is the same. More precisely, if a downstream curve $N_D$ does not deviate much from a straight line with slope $q$ for an extended period of time, and traffic is queued, then curve $N_U$ should become approximately parallel to $N_D$ and remain above $N_D$ by a reproducible number of vehicles, $m_{UD}(q)$. This separation is the average accumulation of vehicles between observers “U” and “D”. We will say that the stationary reproducibility postulate holds approximately if the $N_U$ and $N_D$ curves fluctuate within reasonable bounds about parallel trendlines that are $m_{UD}(q)$ vehicles apart, and if this separation only depends on the slope $q$. The separation should not depend on anything else; e.g., the history of the system. Note that the postulate can be satisfied approximately even if there are stop-and-go oscillations, and that it could be useful (if proven true) even if the remaining postulates of the KW theory are not accurate.

This postulate is also interesting because it applies to inhomogeneous road sections and does not require that one define a “density” for every point on the road. Instead, vehicle accumulation (an observable number with no ambiguity) becomes the fundamental variable to be predicted. In what follows, the relationship $m_{UD}(q)$ will replace the "fundamental diagram" of KW theory. An example of such a curve is shown in Figure 4a.

If one believes that on a particular road the accumulation $m_{UD}(q)$ between any two points U and D only depends on these points through the distance that separates them and one also believes that the dependence is proportional (i.e., the road is homogeneous), then there may be some merit in normalizing the accumulation-flow relation by distance and writing $\kappa(q) = m_{UD}(q)/L_{UD}$ for the resulting function of $q$. This normalized vertical separation, $\kappa(q)$, has units of “density”, but will be called here the normalized average
accumulation to avoid confusion with the various possible definitions of density.\footnote{The reader can verify that the normalized average accumulation is simply Edie’s (1963) generalized definition of density for the time-space rectangle describing the intervening space between the observation points for the (long) time interval during which the system is stationary. This quantity is unambiguously defined even if the two observation points are so close that a single vehicle doesn’t fit in between.}

The second postulate states that the transition between two neighboring stationary states propagates as a wave from D to U. This is depicted in Figure 4b, where the transitions at U and D are idealized by breakpoints in the N-curves.\footnote{In the KW theory, some transitions tend to spread and in those cases the corners of the N-curves become smoother as the wave moves upstream. This complication can be captured by means of a third (stability) postulate, as explained in Daganzo (1997). Because corner effects do not change the predictions significantly when the stationary states persist for a long time, as will be the case in this paper, the third postulate is not introduced here.}

If the slopes \((q_1, q_2)\) and the separations \((m_{UD}(q_1), m_{UD}(q_2))\) between the two sets of parallel lines are given, we see from the geometry of the figure that the “N-vector” that points from one breakpoint to the other must also be given. Simple geometric considerations reveal that the dimensions of the horizontal and vertical components of this vector, \(w_{12}\) and \(n_{12}\), are related to the slopes and separations by: \(w_{12} = -m/ q\) (where \(m = m_{UD}(q_1) - m_{UD}(q_2)\) and \(q = q_1 - q_2\)) and \(n_{12} = m_{UD}(q_1) + q_1 w_{12}\). The time-component of the N-vector represents the wave trip time, and the count-component the number of vehicles that encounter the wave between locations U and D.

These two quantities have simple graphical interpretations on an accumulation-flow plane, such as Fig. 4a, that contains the two stationary states. Consideration of this figure shows that \(w_{12}\) is the negative slope of the line connecting the two state-points and \(n_{12}\) is the intercept of said line with the accumulation axis.\footnote{The reader can verify that the normalized average accumulation is simply Edie’s (1963) generalized definition of density for the time-space rectangle describing the intervening space between the observation points for the (long) time interval during which the system is stationary. This quantity is unambiguously defined even if the two observation points are so close that a single vehicle doesn’t fit in between.}

In the special case where the \(m_{UD}(q)\) relationship is linear, we see from Figure 4a that the coordinates of the N-vector are independent of the two states. This means that all the N-vectors of a piecewise linear curve \(N_D\) with multiple breakpoints must be identical and that the \(N_U\) curve is therefore an exact translation of the \(N_D\) curve in its totality, as originally noted in Newell (1993).

Procedures based on the above-mentioned ideas will be used in the next section to determine the \(m_{UD}(q)\) curves that best fit the data observed on one day at the Smilowitz et al. (1998) site, and then to predict the N-curves at the same site on a different day.
3 APPLICATION TO EXPERIMENTAL DATA

In order to apply the methodology of Section 2, two input curves are required: a piecewise linear approximation of the downstream-most observer, $N_D$, and an $m_{UD}(q)$ relationship. Both input curves were obtained from the data set mentioned in the introduction. Recall that it contains detailed data for eight locations along a congested single lane of a two-lane road for two days of observation.

To control the statistical degrees of freedom in this application, data from the first day were used to calibrate an accumulation-flow relationship, and this relationship was then used with the $N_D$ curve from the second day to predict the upstream $N_U$ curves. The process was then repeated using the second day’s data for calibration, and the first day’s data for prediction. These tests should help determine if time-dependent vehicle accumulations can be predicted from day to day.

*Constructing $N_D*

The downstream-most observer in the experiment was located 246 feet upstream of a vehicle-actuated traffic signal. Thus, the $N$-curves constructed from data collected at observer 8, $N(8,t)$, exhibited the cyclic pulses of the traffic signal. A piecewise linear approximation of $N(8,t)$ that averaged out these pulses was then constructed using as few breakpoints in the curve as possible while ensuring that the maximum separation between the true and approximated curves remained within a reasonable tolerance. The design tolerance chosen for our study was twenty vehicles. This set-up allowed us to create intervals of stationary flow that were long relative to the wave trip time so as to ensure that our two simple postulates suffice to describe the KW solution (see footnote 8).

Breakpoints were determined by visual inspection of the trend changes in $N(8,t)$. In order to stay within the design tolerance, seven and eight breakpoints were used on the first and second days, respectively. The resulting tolerances were sixteen vehicles on the

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9 Note the similarity with the interpretation of the same variables in Newell (1993).
first day and nineteen on the second day. Our procedure was visual, but the piecewise linear approximation could also be constructed using a mathematical program to select the coordinates of the breakpoints that would minimize the deviations of the linear approximation from the true $N(8,t)$. This may be useful in practical settings where the procedure may have to be repeated many times.

**Accumulation-flow relationship**

To obtain a relationship between stationary accumulation and flow, both the average flows at the downstream-most observer and average accumulation of vehicles between the observers were measured. The average flows were taken to be the slopes of the segments in the piecewise linear approximation described above. The average accumulations were simply the average vertical separations between each segment of the piecewise linear curve and the corresponding portions of the curves recorded by the other observers.

The average accumulations, given by the average vertical separation between curves, were measured only during periods of stationary flow. To estimate such accumulations, each linear section of the approximation of $N(8,t)$ was translated upwards toward a target curve until the deviations from the target were minimized. Our estimate is then the vertical separation between the original and translated segments.

As an illustration of this procedure, Figure 5 presents the approximation of $N(7,t)$ obtained from one of the segments of $N(8,t)$ on day 1. Since changes in flow were not instantaneously felt by upstream traffic, the average flow segment for a given interval was first shifted to the right slightly by an approximate wave trip time and then upwards. The wave trip time was estimated using a wave speed of 12.5 mph. Since the stationary flow intervals were long relative to the wave trip times (see Figure 5), the results are not sensitive to the particular wave trip time used. (Note the small size of this shift, i.e. segment AB, relative to the length of interval.) The vertical shift (segment BC) minimizes the vertical deviations between $N(7,t)$ and the shifted segment. Note that this shift is *not* the average vehicle accumulation; as shown in Figure 5, the average vehicle accumulation (i.e. the vertical separation between the two parallel linear segments) is
always smaller than this shift.

For each period of stationary flow, the average vehicle accumulations between the downstream-most observer, \(N(8,t)\), and subsequent queued upstream observers were measured with this procedure. As a result, accumulation-flow data points were obtained for each pair of observers (i.e., observers 7 and 8, observers 6 and 8, etc.). The resulting data set contains seven or fewer data points for each pair of observers. Although each observation represented the state of the system for an extended period of time, we felt that there was not enough information in these data to estimate reliably a separate \(m_{UD}(q)\) curve for each pair of observers. This was true in particular for the observers farthest upstream because the queue only reached these locations for brief intervals of time and this resulted in even fewer data points. In view of this, the site was initially treated as a homogeneous highway so that accumulation could be normalized and then pooled and compared for all observers combined. We shall see later that the homogeneity hypothesis did not hold for all the stretches of our road.

The normalized accumulation-flow data points are plotted in Figure 6. Unlike conventional flow-density diagrams, Figures 6 and 8 display flow on the x-axis and normalized accumulation on the y-axis. This is our way of stressing that flow is the main determinant of vehicle accumulations in queued traffic.

Data points including many vehicles (e.g. those arising from long stationary intervals) are likely to yield more accurate average accumulations than data points including fewer vehicles. Likewise, data points where individual vehicles are observed for a long time (e.g. those corresponding to pairs of observers located far apart) are also likely to be more reliable. Therefore, to capture both of these effects, our observations were weighted by the total vehicle-hours included in each data point. The sizes of the circles in Figure 6 reflect this weighting. This is roughly analogous to weighting data points by the inverse of their variances. Although one could argue for other weightings, we note that the observations line up well and that a fitted curve would not be significantly affected by the weighting.

Because the nature of the normalized accumulation-flow relationship was not
known a priori, both a piecewise linear line and a single line were constructed for these data points. As Figure 6 shows, both approaches produced similar results.

**Predicting $N_U$**
The N-curves for queued locations upstream of $N(8,t)$ on day 2 were estimated using the methodology described in section 2. Given the flow in each section of the piecewise linear approximation of $N(8,t)$ for day 2, the normalized accumulation of vehicles between observers was obtained from the $\kappa(q)$ relationship developed from the day 1 data (Figures 6a and b). These normalized accumulations were multiplied by the distances between observers to obtain the upward shifts that were applied to the various segments of $N(8,t)$ to construct the upstream N-curves as shown in Fig. 4b. Because the N-curves obtained with the two $\kappa(q)$ curves in Figure 6a and Figure 6b only differed by a few vehicles when the curve separation was greatest, the predicted N-curves are only presented for the linear case of Figure 6a.

These results are displayed in Figs. 7a-c. The dark lines in these figures are the predictions and the light curves are the true observations. Dark lines are not included where traffic was not queued. The numbers across the top of each figure are the observer numbers corresponding to the N-curves. Note with the exception of $N(3,t)$ the closeness of the true and predicted curves, which suggests that the methodology worked well for most observers. The results are discussed in more detail in the next section.

The process was then repeated using day 2 data to calibrate two $\kappa(q)$ relationships (Figure 8a and b) and then using these relationships to reconstruct the day 1 N-curves. The estimated $\kappa(q)$ relationships are shown in Figs. 8a and 8b, and the predictions for day 1 obtained with the linear $\kappa(q)$ relation of Fig. 8a in Figs. 9a-c. As in the previous case, the predictions based on the non-linear $\kappa(q)$ were similar and are not presented. Note that the results are qualitatively similar to those previously obtained for day 2.

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10 These seven data points correspond to the seven intervals of stationary flow.
11 The validity of this statement can be easily verified without constructing the curves; see footnote 6.
12 The linear segments for $N(2,t)$ are not displayed because this observer was rarely queued. The (short)
4 ANALYSIS

Normalized Accumulation-Flow Relationship

Closer analysis of the normalized accumulation-flow plots highlighted several interesting points. Although individual $\kappa(q)$ points recorded for the first day of observation spanned a relatively narrow range of flows, the linear relationship between normalized accumulation and flow that resulted (Figure 6a) was similar to that produced with data from the second day (Figure 8a) for the full range of flows. In fact, a comparison of Figs. 6a and 8a reveals that the two lines coincide for the highest accumulations (low flows) observed and only diverge by 7 vehs/mile for the lowest accumulation (high flows). The slopes of the two lines are also similar, yielding wave speed estimates of 10.7 MPH (Fig. 6a) and 11.7 MPH (Fig. 8a).

Although the two piecewise linear approximation of the $\kappa(q)$ relationship (Figures 6b and 8b) were close to each other, it is apparent from the figures that they did not curve in the same way on both days. Therefore, our data did not suggest that $\kappa(q)$ was significantly curved in the range of flows observed. In any case, we have already stated that the small (possibly spurious) curvatures that arose each day could not and did not have a significant effect on the accumulation predictions.

Prediction of N-Curves

As shown in Figures 7 and 9, the predicted N-curves for locations within the first mile upstream of the reference location (i.e. curves 4, 5, 6 and 7, upstream of 8) lay on or very near the true N-curves. At these locations, discrepancies between predicted and observed N-curves were not, in general, greater than the deviations between the piecewise linear approximation of $N(8,t)$ and the true $N(8,t)$. Table 1 summarizes the maximum error in the prediction of the N-curves for observers 4 through 7 on both days.\(^{13}\) Only in one

\(^{13}\)Some average flows observed on the second day were outside the range of average flows observed on the first day. Therefore, some of the normalized accumulations predicted for the second day were based on extrapolated data. Despite this difficulty, the predictions for the second day are quite accurate. This

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N(2,t) segments that arise also lie above the true curve.
instance did the error exceed that of the input $N(8,t)$. In addition, we also see from Figures 7 and 9 that for the most part the predicted and observed curves remained within ten vehicles of each other, even for the most distant of the 4 observers, and that accuracy did not deteriorate significantly with distance. Furthermore, when larger fluctuations did appear, these fluctuations did not prevent the predicted line from reapproaching the observed line in later intervals.

<table>
<thead>
<tr>
<th>N-Curve</th>
<th>Maximum Deviation (number of vehicles)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Day</td>
</tr>
<tr>
<td><strong>Input</strong></td>
<td></td>
</tr>
<tr>
<td>$N(8,t)$</td>
<td>16 vehicles</td>
</tr>
<tr>
<td><strong>Predicted Curves</strong></td>
<td></td>
</tr>
<tr>
<td>$N(7,t)$</td>
<td>&lt; 16 vehicles</td>
</tr>
<tr>
<td>$N(6,t)$</td>
<td>19 vehicles</td>
</tr>
<tr>
<td>$N(5,t)$</td>
<td>&lt; 16 vehicles</td>
</tr>
<tr>
<td>$N(4,t)$</td>
<td>&lt; 16 vehicles</td>
</tr>
</tbody>
</table>

Table 1: Maximum Error in Prediction of N-Curves (measured in number of vehicles)

As marked in Figures 7 and 9, there was some over-prediction for the third observer $N(3,t)$ during most intervals when the queue reached that observer on both days. This means that for a given queue discharge rate drivers spaced themselves more widely (and traveled faster) upstream of observer “4” than downstream. This observation could be explained in several possible ways: (i) distance measurement errors, (ii) a possible “end-of-the queue” effect, if drivers were to behave differently when approaching a queue, (iii) a location-specific effect such as an inhomogeneity in the road, or (iv) failure of the theory. We dismissed (i) because the careful distance measurements in Smilowitz et al. (1998) were reconfirmed on another visit to the site. We also dismissed (ii) because $N(4,t)$ was not overpredicted in the same manner when the queue reached only to the fourth observer. On the other hand, we considered (iii) seriously because the over-prediction of $N(3,t)$ occurred on both days. Although observation of the “4-3” road segment revealed that it includes a change in grade, we did not conclude that this is the cause of the problem because wider spacings were also observed between observers “3” suggests that the estimation methodology is robust.
and “2”, and because other explanations are also possible. (Since this road is used by commuters who must be aware that their wait cannot be changed by driving aggressively in the queue, it is also possible that they may have been allowing themselves more comfortable spacings at locations far upstream of the bottleneck, quite independently of the roadway geometry.) We do not speculate further about this issue here because the cause of the over-prediction may not be needed to predict accumulations; e.g. if we find a recipe that is reasonably accurate despite inhomogeneous traffic behavior. This possibility is examined below.

To this end, a separate $\kappa(q)$ relationship was derived for observer 3 using only data from the first day. This was done by plotting only the accumulations between the downstream-most observer and observer 3 on day 1 and fitting a straight line through these points. The resulting relationship was then used to construct the predicted $N(3,t)$ curve for the second day in the usual way. Figure 10 presents the result. The figure only includes the time interval from 7:20 am to 8:10 am because traffic was only queued at observer 3 inside this interval. The improved prediction supports the theory that traffic back-ups can indeed be estimated with the KW theory despite location-specific traffic behavior.

Predictions of $N(3,t)$ for day 1 could not be similarly tested because there were not enough data points from day 2 to estimate a separate $\kappa(q)$ curve for observer 3. Predictions of $N(2,t)$ could not be tested at all in a similar way because the queue only reached location “2” on one of the days.

In any case, the results strongly suggest that it is possible to predict N-curves quite accurately over distances comparable with one mile and for time periods encompassing several hours without the need for calibrating a model on the day of the predictions.14

Visual inspection of the results showed good prediction of the transition between states. The most striking way to see this is by imagining that each segment of a predicted

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14Data from the two days in our data set were not completely independent since, as discussed in Smilowitz et al (1998), day 2 N-curves were adjusted to account for clock errors and these minor adjustments were based on free flow conditions observed on day 1. It should be stressed, however, that the impact of these adjustments on our results are minimal because free flow conditions change only by a small amount from day to day. The errors, which are imperceptible to the eye, were estimated to be at most a few seconds.
N-curve is introduced in sequence as one steps through time, and to note what would happen to the figure if the introduction of a new segment was delayed or omitted. The effect is shown particularly well in Figure 7c. Note how just as soon as the true and predicted N-curves diverged by an amount greater than the design tolerance, another segment was introduced into the solution. If the transition from one stationary state to another had not occurred close to that moment, the discrepancy between predicted and observed N-curves would have grown too large. This indicates that the theory works similarly well during the transitions between states as it does during periods of stationary flow. Note in particular that the change in trend between states (positive or negative) seems to propagate fairly cleanly and sharply from observer to observer, in agreement with postulate (#2).

Queue Formation and Dissipation
Although this is not the main focus of this paper, Figs. 7, 9 and 10 also shed some light on the queue formation and dissipation process.

An N-curve at a location “j” that has undergone several episodes of queued and unqueued traffic can be constructed by taking the lower envelope of the predicted (queued) curve from N(8,t), obtained as described above, and the curve obtained by shifting N(1,t) to the right by the “free-flow” trip time from “1” to “j”. This is the simplified KW recipe recommended in Newell (1993). In other words, if the simplified KW theory holds then the shifted N(1,t) curves should be above and to the left of the shifted queued curves obtained from N(8,t) when traffic is queued, and they should be below and to the right at other times. The reader can easily verify from Figs. 7, 9 and 10 that this is the case (approximately) with our data.

Single Shift Method
As mentioned earlier, the predictions did not change much with the piecewise linear κ(q) curves; therefore a linear relationship between normalized accumulation and flow seems reasonable for our site. We have already seen that when this relationship is linear the (piecewise linear) downstream N-curve can be translated as a whole, in a single shift,
upwards and to the right in order to construct any of the upstream N-curves. Because the shift is independent of the piecewise linear approximation used for the N-curve, the procedure can be applied to an N-curve with as many break-points as desired; i.e. it can be applied to the raw data curve. This simplifies matters further.\textsuperscript{15} We will refer to this methodology as the “single shift” method.

This method worked well and a sample of the results are shown in Figure 11.\textsuperscript{16} Over short distances, the predicted N-curves matched the true N-curves well. However, over longer distances, the maximum deviations in predictions appeared to be just slightly larger than the maximum deviations using the linear approximation procedure. This occurs because the KW wave does not correlate well the detailed wiggles in the curves near the bottleneck with those upstream of it, as was qualitatively noted in Smilowitz, et al. (1997). This suggests that the finer details of the N-curves do not propagate as a simple KW wave at our site, although their gross behavior does. Fortunately, it is this gross behavior that is the most important determinant of traffic back-ups.

5 CONCLUSION

The results presented here suggest that, even when queued traffic appears to behave in a manner that is inconsistent with the kinematic wave theory on a fine level of detail, a reproducible relationship between normalized accumulation and flow exists. The results also suggest that it is possible to predict vehicle accumulations and queues approximately on a coarse level of detail.

Cumulative counts inside queues were predicted with errors bounded by an acceptable error in the input data. Error tolerances of sixteen and nineteen vehicles in the input data led to smaller prediction errors with only one exception, despite the long duration of the study. We also observed that predictions did not deteriorate appreciably during the transition between states; i.e. that these transitions seem to propagate sharply forward.

\textsuperscript{15} An advantage of the single shift method is that it can be applied without smoothing the data and therefore can be used for real-time predictions.
\textsuperscript{16} Corrections for inhomogeneity in the road between observers 3 and 4 were incorporated in this application as well.
through the traffic stream.

Although the research methodology did not require an assumption of a single wave speed, the single wave speed was found to work well. This should not be surprising, however, because of the relatively narrow ranges of flow that arose in our site. Therefore, we are not yet ready to accept this conclusion for more extreme situations (e.g. when traffic is stopped). To explore this further it would be beneficial to consider other sites. Yet, it is reassuring to note that the normalized accumulation measured at this site during the brief episodes when traffic was completely stopped was very close (approximately 5-10% higher) to the intercepts of the extrapolated curves.

To be sure, we still do not know what is it that drivers do to generate some of the patterns observed in our data. In particular, we do not understand the source of the long period oscillations observed at locations 4, 3 and 2 on both days, and whether this is a peculiarity at our site. Thus, it is important to look into this issue further, both, with this data set and at other sites.

It is also important to remember that our test site is a single lane road with no passing. Therefore, the experiment should be repeated with data from multi-lane highways to see which of the phenomena also occur there. However, given the fact that the methodology performed well for this site, where individual drivers can have a more significant impact, it is not unreasonable to expect that time-dependent back-ups can also be predicted similarly well (or perhaps even better) on facilities where passing is possible. We hope that the results in this paper will encourage others to verify or disprove our findings. This is desirable because an ability to predict traffic back-ups is a prerequisite for managing freeway networks effectively.

ACKNOWLEDGEMENTS

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