Robust Platoon Maneuvers for AVHS

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Abstract

The AVHS architecture of the California PATH program organizes traffic into platoons of closely spaced vehicles. Platoons are formed and broken up by two longitudinal control maneuvers, the merge and the split. A third longitudinal maneuver, decelerate to change lanes, allows a platoon switching from one lane to another to enter its new lane at a safe spacing and speed.

The maneuvers, particularly the merge, can be potentially hazardous. In a merge, the cars in the trail platoon are moving faster than those in the lead platoon, while the gap separating the two platoons is smaller than usual. A sudden deceleration by the lead platoon could cause a high-speed collision. If the relative velocities of the merging platoons can be set so that they are guaranteed never to collide at a high relative velocity, the merge can be considered safe. A maximum safe velocity for the trail platoon can be found for any given spacing and lead-platoon velocity.

This paper presents a merge maneuver in which the velocity of the trail platoon never exceeds the maximum safe velocity. The controller switches among several feedback laws that keep the velocity of the trail platoon under a safe velocity profile and within comfort limits on jerk and acceleration, under normal circumstances. This merge maneuver can be considered the fastest merge strategy that does not violate bounds on safety and comfort. The controller is also more robust to changes in the vehicles' acceleration capability than those that use a desired open-loop trajectory.

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The control approach used for the merge maneuver can be applied to the other maneuvers to ensure that they never result in a collision. The switching controllers for the split and decelerate to change lanes maneuvers also yield a more comfortable ride than those that track a timed trajectory.
1 Introduction

The Automated Vehicle/Highway System architecture of the California PATH program organizes traffic into platoons of closely spaced vehicles (Varaiya, Shadlo 1991). The tight spacing between cars within a platoon prevents collisions at high relative velocities. The gaps between platoons are large, ensuring that a platoon will have time to stop even if the platoon ahead of it brakes abruptly.

Platoons can perform three basic maneuvers. In a merge, two platoons join to form one platoon; in a split, one platoon breaks into two; and in a change lane, a platoon of one car switches into an adjacent lane. Platoons of more than one car can not change lanes. Before the change lane maneuver can occur, the single car must be at a safe distance from platoons in the adjacent lane. The decelerate to change lanes maneuver creates this safe spacing.

This paper presents new control strategies for the three longitudinal maneuvers: merge, split and decelerate to change lanes. The previous controller design relied on nominal open-loop trajectories that the platoon executing the maneuver attempted to track (Godbole, Lygeros 1994). The maneuvers were safe and comfortable for passengers under normal circumstances. If, however, the platoon ahead of the one performing the maneuver underwent large accelerations or decelerations, comfort and safety could be compromised. If the acceleration capabilities of the platoon tracking the trajectory were lower than expected, the maneuvers may not have been completed at all.

The control approach presented in this paper is intended to improve the robustness of these maneuvers to such factors as the deceleration of the lead platoon and variable acceleration capability. Robustness is considered here as the ability to complete the maneuvers safely, comfortably and as quickly as possible.

The key to the new controllers is eliminating the timed trajectories. Instead, feedback laws keep the velocity of the platoon in the maneuver within a safety limit. They also keep its jerk and acceleration within comfort boundaries except when safety becomes critical. Completion of the maneuvers in this design does not depend on meeting a desired open-loop acceleration trajectory.

The new merge maneuver prevents even low-speed collisions in all but the most extreme cases of lead platoon deceleration. If the lead platoon applies and holds maximum braking during a merge, a collision could still occur, but the relative velocity at impact will be within an acceptable limit. The new split and decelerate to change lanes maneuvers ensure a comfortable ride in nearly all cases. The cost of improved safety and comfort is performance, which is measured by how quickly the maneuvers are completed.

The controllers can also be used under degraded road or weather conditions by simply changing certain parameters in the control laws, such as the maximum deceleration or the maximum comfortable jerk. Section 2 of the paper reviews the design introduced in (Godbole, Lygeros 1994) for the maneuvers. Section 3 presents the safe velocity profile for the trail platoon in a merge. Section 4 describes a controller that uses a finite-state machine to switch among feedback laws designed so the trail platoon completes the merge as quickly as possible without violating the safety or comfort limits. The feedback laws and state machines
for the other two maneuvers are presented in Section 5. Simulation results are presented in section 6. Equation derivations and a list of all symbols used in the paper are in appendices A through D.

2 Previous Maneuver Design

In the current PATH AHS architecture, an optimal platoon size is set based on the flow and traffic density on a section of highway. If a platoon is smaller than optimal, it will seek to join with the platoon ahead of it. The leader of the trail platoon will ask the leader of the platoon ahead to merge. If the platoon ahead is not busy with another maneuver, and if the resulting platoon will not be too large, its leader will accept the merge request. The leader of the trail platoon will then switch from the standard leader control mode to the merge control mode. This communication process is described fully in (Hsu et al. 1991).

If a platoon is too large, or if a car within the platoon must leave the platoon to switch lanes, a car in the platoon will ask its leader to allow it to split. If the leader is not busy, it will allow the split. The first car in what will be a new platoon then enters the split control mode. A similar type of communication occurs before a car or platoon begins the decelerate to change lanes maneuver.

The current maneuvers are described in (Godbole, Lygeros 1994). Like all operating modes for the platoon leader, the maneuvers assume that the nonlinear car dynamics are feedback linearized so that the resulting system is a triple integrator with a jerk input.

In the merge, a trajectory $\Delta x_{\text{desired}}(t)$ is generated for the trail platoon so that $\Delta x_{\text{desired}}(0)$ and $\Delta \dot{x}_{\text{desired}}(0)$ correspond to the initial spacing and relative velocity between the lead and trail platoons. $\Delta x_{\text{desired}}(t_{\text{final}})$ and $\Delta \dot{x}_{\text{desired}}(t_{\text{final}})$ are the desired intraplatoon spacing and zero relative velocity. The trajectory also defines $\Delta \ddot{x}_{\text{desired}}(t)$ and $\Delta \dddot{x}_{\text{desired}}(t)$, the second and third derivatives of $\Delta x_{\text{desired}}(t)$.

Because the maneuver is designed to be time-optimal, the acceleration trajectory is bang-bang. The bang-bang acceleration curve is smoothed so that changes in acceleration do not violate jerk comfort constraints. Comfortable acceleration limits are $\pm 2m/s^2$, and jerk limits are $\pm 5m/s^3$.

The optimal merge trajectory is set assuming that the lead platoon does not accelerate during the maneuver. Feedback control is used to guarantee asymptotic tracking if the velocity of the lead platoon is not constant. The controller output $u$ is:

$$u = \ddot{x} = \Delta \ddot{x}_{\text{desired}} + k_2(\dot{x} - \Delta \dot{x}_{\text{desired}}) + k_1(\Delta \dot{x} - \Delta \dot{x}_{\text{desired}}) + k_0(\Delta x - \Delta x_{\text{desired}}),$$

(1)

where $\Delta x(t)$ is the gap from the front of the trail platoon to the rear of the lead platoon, and $\Delta \dot{x}(t)$ is its derivative. The acceleration and jerk of the trail platoon are $\dot{x}(t)$ and $\ddot{x}(t)$, respectively. $k_0$, $k_1$ and $k_2$ are the feedback gains. Sensors are assumed to measure $\Delta x$ and $\Delta \dot{x}$. The velocity and acceleration of the trail platoon are also assumed to be available from
sensors. No communication between platoons is assumed, so the lead platoon’s acceleration is not used in the feedback law.

The split maneuver uses a trajectory that is the opposite of that for the merge. First maximum comfortable deceleration is applied, then maximum comfortable acceleration. The control law is the same.

The decelerate to change lanes maneuver helps a car changing lanes enter the new lane at a safe spacing and velocity. If the car changing lanes senses a platoon in the adjacent lane at an unsafe spacing, either the car or platoon will slow to create a safe gap. An imaginary platoon is assumed in the same lane as the vehicle executing the maneuver. The imaginary platoon is at the same spacing and velocity as the platoon in the other lane. The vehicle doing the maneuver splits from the imaginary platoon.

Because these maneuvers were designed to be completed as quickly as possible, safety and comfort can be compromised in certain circumstances.

In a merge from 30 m initial spacing, the trail platoon reaches a relative velocity of 8 m/s when it is about 15 m behind the lead platoon. If the lead platoon were to apply maximum braking at that moment, the trail platoon would hit it two seconds later at a relative velocity of at least 8 m/s. These high-speed collisions have been observed in simulations using SmartPath, a program that simulates operation of an entire automated highway (Eskafi, Varaiya 1992).

Failures could also create hazards during a merge. In (Hitchcock 1994), Hitchcock argues that it is safer to apply full braking when a car’s longitudinal controller fails than to apply no braking. He finds that such a failure is more than 10 times as likely to produce a fatality or serious injury if the failure occurs in the lead platoon during a merge than if it occurs during normal operation.

The desired trajectory in these maneuvers is an explicit function of time. The maneuvers must be completed by \( t = t_{\text{final}} \). The design can then only guarantee that the relative acceleration between the lead and trail platoons satisfies the comfort limits. If the lead platoon decelerates while the trail platoon is tracking the maximum comfortable deceleration, the trail platoon will have to decelerate further to stay on the trajectory, creating an uncomfortable ride.

The timed trajectories also require that the platoon accelerate at a given level. Cars’ performance capabilities change with such factors as load and road grade. A platoon may not be able to track the predetermined acceleration trajectory. If that happens, the platoon may not complete the maneuver in the time allotted, or, in a merge, the platoon may accelerate for most of the maneuver and then brake hard at the end. Comfort limits on jerk and deceleration would not be maintained.
3 Safe Merge Strategy

3.1 Safe Curve Derivation

In a safe merge, the impact velocity in a collision should never exceed an acceptable limit. High-speed collisions are most likely to occur if the lead platoon applies and holds maximum braking while the merge is in progress. The maximum safe approach velocity for the trail platoon is then defined as the top speed that would allow it to hit the lead platoon at the acceptable impact velocity if the platoon ahead were to apply and hold full braking.

The maximum safe velocity is a function of the spacing between platoons and the velocity of the lead platoon. The curve defining the safe velocity consists of two sections. In the first section, the trail platoon is far enough from the lead platoon that maximum deceleration will stop the lead platoon before the trail platoon hits it. The safe velocity curve in this section is:

\[ v_{safe1}(v_{lead}(t), \Delta \tau(t)) = -a_{min}d + v_{lead}(t)^2 + v_{allow}^2 + 2a_{min}\Delta \tau(t) \]  

(2)

The terms are defined as follows:

- \( v_{safe1} \) is the first portion of the maximum safe velocity curve.
- \( a_{min} \) is the absolute value of the maximum emergency deceleration.
- \( v_{lead} \) is the velocity of the lead platoon. It can be determined by adding the signals from the relative velocity sensor and the onboard velocity sensor in the lead car of the trail platoon.
- \( v_{allow} \) is the allowable relative impact velocity.
- \( \Delta \tau \) is the spacing between the rear of the lead platoon and the front of the trail platoon.
- \( d \) is any delay between the time when the lead platoon applies maximum braking and the time when the trail platoon does.

The derivation of this equation is shown in Appendix A. A similar approach curve was defined in (Sklar et al. 1979).

If full braking does not stop the lead platoon before the trail platoon hits it, the safe velocity is given by:

\[ v_{safe2}(v_{lead}(t)) = -a_{min}d + v_{lead}(t) + v_{allow} \]  

(3)

where \( v_{safe2} \) is the second portion of the safe velocity curve.

In a safe, time-optimal merge, the trail platoon tracks the maximum safe velocity curve defined by these equations. The platoon's initial and final velocities are usually well under the safe curve. To reach the safe curve, the trail platoon can accelerate at the maximum comfortable level, or maximum possible level, to start the maneuver. To slow to \( v_{lead} \) at the end of the merge, the trail platoon decelerates at the maximum comfortable level. The deceleration curve can be written as a function of \( v_{lead} \) and \( \Delta \tau \).
Figure 1: Basic velocity profile for 60 m initial spacing. The lead platoon is moving at a constant velocity of 25 m/s.

\[ v_{\text{decel}}(v_{\text{lead}}(t), \Delta x(t)) = v_{\text{lead}}(t) + \sqrt{2a_{\text{com}}(\Delta x(t) - \Delta x_{\text{desired}})} \]  

(4)

New terms are defined as:

- \( a_{\text{com}} \) is the absolute value of the maximum comfortable acceleration and deceleration.
- \( \Delta x_{\text{desired}} \) is the desired intraplatoon spacing.

The derivation of this equation is also presented in Appendix A.

The acceleration portion of the trail platoon’s trajectory can not be easily expressed in terms of \( \Delta x \) and \( v_{\text{lead}} \). The initial relative velocity and relative acceleration of the two platoons are variable, as is the acceleration capability of the cars in the trail platoon. Safety is least critical in this region, however, and a desired velocity need not be defined. The desired acceleration is given by:

\[ \dot{v}_{\text{accel}} = a_{\text{com}} \]  

(5)

At highway speeds, most cars can not achieve \( a_{\text{com}} \), which is taken to be 2 \( m/s^2 \) (Gillespie 1992). The maximum acceleration a car can reach is a function of its velocity and will be denoted as \( a_{\text{max}}(v) \).

Figure 1 shows the basic desired velocity profile of the trail platoon in the phase plane during a safe merge. This profile is only for the case of constant lead platoon velocity.
3.2 Blending

The phase-plane trajectory in figure 1 includes abrupt changes in acceleration at each point where curves intersect and at the beginning and end of the maneuver. These sections must be smoothed so as not to violate the jerk comfort constraints. The following changes are made in the basic profile to smooth the changes in acceleration.

- Maximum comfortable jerk must be applied at the end of the maneuver to bring the final acceleration of the trail platoon equal to that of the platoon ahead. The deceleration curve should bring the trail platoon to a spacing and velocity at which applying the jerk comfort limit will bring it to the final desired spacing and velocity. This is accomplished by changing the deceleration equation to

\[
v_{\text{decel}}(v_{\text{lead}}(t), \Delta x(t)) = v_{\text{lead}}(t) + \frac{\sqrt{a_{\text{com}}^4 + 2a_{\text{com}}(\Delta x(t) - \Delta x_{\text{desired}} - \frac{a_{\text{com}}^2}{6j_{\text{com}}^2})}}{a_{\text{com}}^2}
\]

\(j_{\text{com}}\) is the absolute value of the maximum comfortable jerk. These modifications are described in Appendix A.

- To make the transition from the acceleration curve to the first part of the safe velocity curve, \(-j_{\text{com}}\) must be applied at the correct point on the acceleration curve. Acceleration along the first part of the safe velocity curve can be estimated as \(a_{\text{lead}} - \frac{a_{\text{com}}}{2}\). The total change in acceleration from the acceleration curve to the safe curve is then \(a_{\text{lead}} - a_{\text{trail}} - \frac{a_{\text{com}}}{2}\). \(a_{\text{lead}}\) and \(a_{\text{trail}}\) are the accelerations of the lead and trail platoons. Assuming that the maximum comfortable jerk is used, the time required to make this change in acceleration is \((.5a_{\text{com}}/2 + a_{\text{trail}} - a_{\text{lead}})/j_{\text{com}}\). The change in velocity during that length of time will be \((a_{\text{trail}}^2 - (.5a_{\text{com}} - a_{\text{lead}})^2)/2j_{\text{com}}\).

Define \(v_{\text{ahead}}, v_{\text{ahead1}}\) and \(\Delta x_{\text{ahead1}}\) as the presumed trail and lead platoon velocities and relative spacing, respectively, at time \(t + (.5a_{\text{com}} + a_{\text{trail}} - a_{\text{lead}})/j_{\text{com}}\) if the trail platoon were to apply and hold \(-j_{\text{com}}\) until that time. To end the blend on the safe curve, negative maximum jerk is needed when the velocity given by the safe curve at \((v_{\text{ahead1}}, \Delta x_{\text{ahead1}})\) is \(v_{\text{ahead}}\). The blend begins when the following inequality is satisfied:

\[
v_{\text{safe1}}(v_{\text{ahead1}}(t), \Delta x_{\text{ahead1}}) \leq v_{\text{trail}}(t) + (a_{\text{trail}}^2 - (.5a_{\text{com}} - a_{\text{lead}})^2)/2j_{\text{com}},
\]

where \(v_{\text{trail}}\) is the velocity of the trail platoon.

These calculations are based on the assumption that the jerk of the lead platoon during the blend is zero. To generalize the blend to handle all cases, the jerk of the trail platoon must be \(j_{\text{lead}} - j_{\text{com}}\) during the blend.

- A similar calculation can be made for the transition between the second portion of the safe curve and the deceleration curve. In this case, the time in the blend is \(a_{\text{com}}/j_{\text{com}}\).
\( \Delta x_{\text{ahead}3} \) and \( v_{\text{ahead}3} \) are the spacing and lead platoon velocity at time \( t + a_{\text{com}} / j_{\text{com}} \). The change in trail platoon velocity is \((2a_{\text{trail}}a_{\text{com}} - a_{\text{com}}^2)/2j_{\text{com}}\). Negative maximum jerk should be applied when

\[
v_{\text{decel}}(v_{\text{lead}}(t), \Delta x_{\text{ahead}3}) \leq v_{\text{trail}}(t) + (2a_{\text{trail}}a_{\text{com}} - a_{\text{com}}^2)/2j_{\text{com}}. \tag{8}
\]

In a merge from a short distance, the trail platoon may have to make a transition from the acceleration curve directly to the deceleration curve. \( \Delta x_{\text{ahead}2} \) and \( v_{\text{ahead}2} \) in this case are the spacing and lead platoon velocity at \( t + (a_{\text{trail}} + a_{\text{com}} - a_{\text{lead}})/j_{\text{com}} \). The velocity change is \((a_{\text{trail}}^2 - (a_{\text{com}} - a_{\text{lead}})^2)/2j_{\text{com}}\). The condition for applying negative maximum jerk is:

\[
v_{\text{decel}}(v_{\text{lead}}(t), \Delta x_{\text{ahead}2}) \leq v_{\text{trail}}(t) + (a_{\text{trail}}^2 - (a_{\text{com}} - a_{\text{lead}})^2)/2j_{\text{com}} \tag{9}
\]

Transitions from the first portion of the safe velocity curve to the deceleration curve are also possible. Because the acceleration on that curve is negative, applying maximum negative jerk when the condition in equation (8) is valid will ensure that the trail platoon’s velocity remains below the deceleration curve. Feedback control will eventually bring the velocity onto the curve.

- No special blend is needed for the transition between the two portions of the safe curve or at the start of the maneuver. The acceleration on the second portion is greater than that on the first. By constraining any positive jerk to remain under the comfort level, the velocity will drop below \( v_{\text{safe}2} \) and gradually increase to it. The saturation function can also be applied at the start of the merge when the acceleration is increasing to \( a_{\text{com}} \).

- One additional change is made in the velocity profile. To allow for disturbances and unaccounted for delays in the controller, the actual safe velocity profile that is tracked is slightly below the profile defined above. Adding this \( v_{\text{buffer}} \) terms yields the equations:

\[
v_{\text{safe}1}(v_{\text{lead}}(t), \Delta x(t)) = -a_{\min}d - v_{\text{buffer}} + \sqrt{v_{\text{lead}}(t)^2 + v_{\text{allow}}^2 + 2a_{\min} \Delta x(t)} \tag{10}
\]

\[
v_{\text{safe}2}(v_{\text{lead}}(t)) = -a_{\min}d + v_{\text{lead}}(t) + v_{\text{allow}} - v_{\text{buffer}}. \tag{11}
\]

A velocity profile with these modifications is compared to the original profile in figure 2.

4 Merge Control

As in (Godbole, Lygeros 1994), feedback linearization is assumed in designing the controller for the safe merge. The control output is the jerk. Each section of the trajectory described
above, including blending regions, has a feedback law associated with it. The controller
switches among these feedback laws depending on the current values of \(v_{\text{trail}}, v_{\text{lead}}\) and \(\Delta x\).

The controller can be modeled as a finite-state machine. Each state corresponds to one
of the regions of the profile described above, such as the acceleration curve or the blend
between it and the safe velocity curve. Except for the acceleration curve, the curves in the
velocity profile and conditions to begin a blend define two-dimensional boundaries in the
space of \((\Delta x, v_{\text{lead}}, v_{\text{trail}})\). Transitions occur when one of the boundaries is crossed. The
state corresponding to the acceleration curve does not have to be clearly defined in this
space because it is the initial state and no transitions are made into it.

A state machine indicating the possible transitions is given in figure 3. Table 1 lists the
conditions under which transitions are taken.

The feedback law for each state is described below.

- \(v_{\text{safe1}}, v_{\text{safe2}}, \) and \(v_{\text{decel}}\). The control law takes the same form in all of these states:

\[
    u = \ddot{\bar{x}} = \ddot{x}_{\text{desired}} + k_a (\ddot{x}_{\text{desired}} - \ddot{x}) + k_v (\dddot{x}_{\text{desired}} - \dddot{x})
\]  

(12)

In these laws, \(\dddot{x}_{\text{desired}}\) is given by equations (10), (11) and (6) for the states \(v_{\text{safe1}}, v_{\text{safe2}}, \) and \(v_{\text{decel}},\) respectively. The \(\dddot{x}_{\text{desired}}\) and \(\dddot{x}_{\text{desired}}\) terms in the control laws are
the first and second time derivatives of the desired velocity. These derivatives are given by:

\[
    \dot{v}_{\text{desired}} = \frac{\delta v_{\text{desired}}}{\delta \Delta x} \Delta x + \frac{\delta v_{\text{desired}}}{\delta v_{\text{lead}}} v_{\text{lead}}
\]  

(13)
Figure 3: Finite-state machine model of controller. The states are listed in bold. The edge labels correspond to the conditions listed in table 1.
<table>
<thead>
<tr>
<th>Edge</th>
<th>Begin state</th>
<th>To state</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>$v_{\text{accel}}$</td>
<td>blend$_1$</td>
<td>$v_{\text{trail}} \geq v_{\text{safe}<em>1}(v</em>{\text{ahead}<em>1}, \Delta x</em>{\text{ahead}<em>1}) - \frac{(a</em>{\text{trail}} - (0.5a_{\text{com}} - a_{\text{lead}})^2)}{2j_{\text{com}}}$</td>
</tr>
<tr>
<td>a2</td>
<td>$v_{\text{accel}}$</td>
<td>blend$_2$</td>
<td>$v_{\text{trail}} \geq v_{\text{decel}}(v_{\text{ahead}<em>2}, \Delta x</em>{\text{ahead}<em>2}) - \frac{(a</em>{\text{trail}} - (a_{\text{com}} - a_{\text{lead}})^2)}{2j_{\text{com}}}$</td>
</tr>
<tr>
<td>b11</td>
<td>blend$_1$</td>
<td>$\text{safe}_1$</td>
<td>$\Delta x &lt; \Delta x_{\text{ahead}_1}$</td>
</tr>
<tr>
<td>b12</td>
<td>blend$_1$</td>
<td>unsafe</td>
<td>$v_{\text{trail}} \geq v_{\text{safe}<em>1}(v</em>{\text{lead}}, \Delta x)$</td>
</tr>
<tr>
<td>b13</td>
<td>blend$_1$</td>
<td>blend$_2$</td>
<td>$v_{\text{trail}} \geq v_{\text{decel}}(v_{\text{ahead}<em>2}, \Delta x</em>{\text{ahead}<em>2}) - \frac{(a</em>{\text{trail}} - (a_{\text{com}} - a_{\text{lead}})^2)}{2j_{\text{com}}}$</td>
</tr>
<tr>
<td>s11</td>
<td>$\text{safe}_1$</td>
<td>$\text{safe}_2$</td>
<td>$v_{\text{trail}} \leq v_{\text{safe}<em>2}(v</em>{\text{lead}})$</td>
</tr>
<tr>
<td>s12</td>
<td>$\text{safe}_1$</td>
<td>unsafe</td>
<td>$v_{\text{trail}} \geq v_{\text{safe}<em>1}(v</em>{\text{lead}}, \Delta x)$</td>
</tr>
<tr>
<td>s13</td>
<td>$\text{safe}_1$</td>
<td>blend$_2$</td>
<td>$v_{\text{trail}} \geq v_{\text{decel}}(v_{\text{ahead}<em>2}, \Delta x</em>{\text{ahead}<em>2}) - \frac{(2a</em>{\text{trail}}a_{\text{com}} - a_{\text{com}}^2)}{2j_{\text{com}}}$</td>
</tr>
<tr>
<td>s14</td>
<td>$\text{safe}_1$</td>
<td>finish</td>
<td>$\Delta x \leq \Delta x_{\text{desired}} + \frac{a_{\text{com}}^3}{6j_{\text{com}}^2}$</td>
</tr>
<tr>
<td>s21</td>
<td>$\text{safe}_2$</td>
<td>unsafe</td>
<td>$v_{\text{trail}} \geq v_{\text{safe}<em>2}(v</em>{\text{lead}})$</td>
</tr>
<tr>
<td>s22</td>
<td>$\text{safe}_2$</td>
<td>blend$_2$</td>
<td>$v_{\text{trail}} \geq v_{\text{decel}}(v_{\text{ahead}<em>2}, \Delta x</em>{\text{ahead}<em>2}) - \frac{(2a</em>{\text{trail}}a_{\text{com}} - a_{\text{com}}^2)}{2j_{\text{com}}}$</td>
</tr>
<tr>
<td>b21</td>
<td>blend$_2$</td>
<td>$v_{\text{decel}}$</td>
<td>$\Delta x \leq \Delta x_{\text{ahead}}$</td>
</tr>
<tr>
<td>b22</td>
<td>blend$_2$</td>
<td>unsafe</td>
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<tr>
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<td>finish</td>
<td>$\Delta x \leq \Delta x_{\text{desired}} + \frac{a_{\text{com}}^3}{6j_{\text{com}}^2}$</td>
</tr>
<tr>
<td>d1</td>
<td>$v_{\text{decel}}$</td>
<td>unsafe</td>
<td>$v_{\text{trail}} \geq v_{\text{safe}<em>2}(v</em>{\text{lead}})$</td>
</tr>
<tr>
<td>d2</td>
<td>$v_{\text{decel}}$</td>
<td>finish</td>
<td>$\Delta x \leq \Delta x_{\text{desired}} + \frac{a_{\text{com}}^3}{6j_{\text{com}}^2}$</td>
</tr>
<tr>
<td>u1</td>
<td>unsafe</td>
<td>$\text{safe}_1$</td>
<td>$v_{\text{trail}} \leq v_{\text{safe}<em>1}(v</em>{\text{lead}}, \Delta x)$, and $v_{\text{safe}<em>2}(v</em>{\text{lead}}, \Delta x) &gt; v_{\text{safe}<em>2}(v</em>{\text{lead}}, \Delta x)$, and $v_{\text{safe}<em>1}(v</em>{\text{lead}}, \Delta x) &lt; v_{\text{decel}}(v_{\text{lead}}, \Delta x)$</td>
</tr>
<tr>
<td>u2</td>
<td>unsafe</td>
<td>$\text{safe}_2$</td>
<td>$v_{\text{trail}} \leq v_{\text{safe}<em>2}(v</em>{\text{lead}})$ and $v_{\text{safe}<em>2}(v</em>{\text{lead}}, \Delta x) &lt; v_{\text{decel}}(v_{\text{lead}}, \Delta x)$</td>
</tr>
<tr>
<td>u3</td>
<td>unsafe</td>
<td>$v_{\text{decel}}$</td>
<td>$v_{\text{trail}} \leq v_{\text{decel}}(v_{\text{lead}}, \Delta x)$ and $v_{\text{decel}}(v_{\text{lead}}, \Delta x) &lt; v_{\text{safe}<em>2}(v</em>{\text{lead}})$</td>
</tr>
</tbody>
</table>

| $\Delta x_{\text{ahead}_1}$ for $t + (a_{\text{trail}} - a_{\text{lead}} + 0.5a_{\text{com}})/j_{\text{com}}$ |
| $\Delta x_{\text{ahead}_2}$ for $t + (a_{\text{trail}} - a_{\text{lead}} + a_{\text{com}})/j_{\text{com}}$ |
| $\Delta x_{\text{ahead}_3}$ for $t + a_{\text{com}}/j_{\text{com}}$ |

Table 1: Transition table for state machine shown in figure 3
\[
\ddot{v}_{\text{desired}} = \frac{\delta \dot{v}_{\text{desired}}}{\delta \Delta x} \Delta x + \frac{\delta \dot{v}_{\text{desired}}}{\delta v_{\text{lead}}} \dot{v}_{\text{lead}} + \frac{\delta \dot{v}_{\text{desired}}}{\delta v_{\text{trail}}} \dot{v}_{\text{trail}} + \frac{\delta \dot{v}_{\text{desired}}}{\delta v_{\text{lead}}} v_{\text{lead}}
\]  

(14)

Note that these derivatives, which are listed in Appendix B, are functions of the jerk and acceleration of the lead platoon as well as its velocity and the interplatoon spacing. The jerk and acceleration of the lead platoon are not available from sensors. They are calculated by numerically differentiating the velocity signal, which is assumed to be filtered. Any lags caused by filtering or differentiation should be included in the delay term. The differentiation does not have to be precise. Small errors in calculating the acceleration and jerk are overcome by the feedback terms.

- \(v_{\text{accel}}\) A simple first-order control law is used in this state.

\[
u = k_a (\ddot{x}_{\text{desired}} - \ddot{x}) \]

(15)

where \(\ddot{x}_{\text{desired}} = a_{\text{com}}\).

The transitions from this state do not depend on a specific acceleration. If the trail platoon can not accelerate as quickly as desired, the maneuver will take longer, but comfort and safety will not be affected.

- \(\text{blend}_1\) and \(\text{blend}_2\) Each of these states is defined by the section of the velocity profile on which the blend ends. The state \(\text{blend}_2\) includes all blends that end on the \(v_{\text{decel}}\) curve regardless of where they begin. Each blend selects a point on the target section of the profile. That point is at the spacing defined above as \(\Delta x_{\text{ahead}}\). \(\Delta x_{\text{ahead}}\) and \(v_{\text{ahead}}\) define a velocity and an acceleration on the target section. The control law in these states seeks to bring the velocity and acceleration of the trail platoon to the values at the specified point when \(\Delta x = \Delta x_{\text{ahead}}\). Recall that the nominal control that accomplishes this goal is \(\ddot{x}_{\text{lead}} - j_{\text{com}}\). To account for errors, feedback is used. Let

\[
u_1 = \ddot{x} = \ddot{x}_{\text{lead}} + k_a (\ddot{x}_{\text{desired}} - \ddot{x}) + k_a (\ddot{x}_{\text{desired}} - \ddot{x})
\]

(16)

where \(\ddot{x}_{\text{desired}}\) and \(\ddot{x}_{\text{desired}}\) are functions of \(v_{\text{lead}}, \dot{v}_{\text{lead}}\) and \(\Delta x_{\text{ahead}}\). The control law is given by:

\[
u = \begin{cases} 
\nu_1 & \text{if } \nu_1 > -j_{\text{com}} \\
\dot{x}_{\text{lead}} - j_{\text{com}} & \text{otherwise}
\end{cases}
\]

(17)

- \(\text{finish}\) The final blend is completed by using the existing control law for followers in a platoon. Tracking the modified deceleration curve leaves the trail platoon at \(\Delta x = \Delta x_{\text{desired}} + a_{\text{com}}^2/6j_{\text{com}}^2; \dot{x}_{\text{trail}} = v_{\text{lead}} + a_{\text{com}}^2/2j_{\text{com}}; \) and \(\ddot{x}_{\text{trail}} = a_{\text{lead}} - a_{\text{com}}\). Although the trail platoon has not reached the final desired values, at this point the merge ends and the cars in the trail platoon become followers of the lead car in the lead platoon.
The control law for followers in a platoon is described in (Swaroop et al. 1994). Simulations of that controller with the initial conditions noted above indicate the follower law can complete the merge. Ending the merge here can save as much as one second of the time taken by the maneuver, depending on what value of $j_{com}$ is selected. Simulation results for the follower controller are shown in figure 4.

- **unsafe** Whenever the velocity of the trail platoon exceeds the safety boundaries defined by $v_{afe1}$ and $v_{afe2}$, the platoon applies maximum braking. Maximum braking is held until the velocity is below the modified safety limit defined by $v_{afe1}$ and $v_{afe2}$.

$$u = -j_{max}$$

(18)

The value of $v_{buffer}$ is chosen so that a platoon should never enter the **unsafe** state under normal conditions, which include large lead platoon decelerations. If, however, a car cuts in between the two platoon involved in the merge, the **unsafe** state would be entered. Also, if a sensor in the lead car of the trail platoon fails or is less accurate than expected, the **unsafe** state could be entered.

A saturation function helps enforce comfort bounds. Positive jerk is always constrained to remain under the comfort limit. Negative control action is more critical to safety, so
negative jerk is only constrained when the lead platoon’s deceleration is small. Whenever the unsafe state is entered, the limits on jerk are removed.

5 Other Maneuvers

Similar switching controllers can also be used for the split and decelerate to change lanes maneuvers. Because these maneuvers do not present the threat of high-speed collisions in most cases, the time-optimal trajectory used in the current design is desirable.

A bang-bang acceleration profile can be generated using the finite-state machine approach. In one state the desired acceleration is \(-a_{\text{com}}\); in the next, it is \(a_{\text{com}}\), or more likely \(a_{\text{max}}\). These comfort levels can be maintained regardless of what the lead platoon does, unless safety is threatened. A split should never result in a crash, so the safety boundary is more stringent that the one for the merge. Again the boundary is defined for the case in which the lead platoon applies and holds maximum braking. The physical jerk limit of the trail platoon is assumed to be large. The allowable impact velocity is zero.

\[
v_{\text{safe}}(v_{\text{lead}}(t), \Delta x(t)) = -a_{\text{min}}d - v_{\text{buffer}} + \sqrt{v_{\text{lead}}(t)^2 + 2a_{\text{min}}\Delta x(t)}.
\]

A transition must be made from the deceleration state to the acceleration state in the split maneuver. The transition boundary should be represented by a surface in the \((\Delta x, v_{\text{lead}}, v_{\text{trail}})\) space. The following equation defines the transition surface:

\[
v_{\text{accel}}(v_{\text{lead}}(t), \Delta x(t)) = v_{\text{lead}}(t) - \sqrt{2a_{\text{max}}(v_{\text{trail}})(\Delta x_{\text{desired}} - \Delta x(t))},
\]

where \(\Delta x_{\text{desired}}\) is the safe interplatoon spacing. \(a_{\text{max}}(v_{\text{trail}})\) is taken to be less than \(a_{\text{com}}\). Because it is a function of velocity, \(a_{\text{max}}\) must be estimated as the maneuver progresses.

The full state machines, transition tables and control laws for both maneuver are given below.

5.1 Split

The state machine model of the controller for the split maneuver is shown in figure 5. Transitions are listed in table 2.

The states are described as follows:

- **start** This state is most critical for safety. The control law is intended to bring the trail platoon to a velocity below a boundary defined by \(v_{\text{safe}}(v_{\text{lead}}, 0)\). The desired acceleration must be less than the acceleration of the leader platoon,

\[
\ddot{x}_{\text{desired}} = \ddot{x}_{\text{lead}} - a_{\text{com}}.
\]

\[
u = \ddot{x} = \ddot{x}_{\text{lead}} + k_a(\ddot{x}_{\text{desired}} - \ddot{x})
\]
Figure 5: Finite-state machine model of controller for split maneuver. The states are listed in bold. The edge labels correspond to the conditions in Table 2.

<table>
<thead>
<tr>
<th>Edge</th>
<th>Begin state</th>
<th>To state</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>s1</td>
<td>start</td>
<td>v_decel</td>
<td>$v_{trail} &lt; v_{safe0}(v_{lead}, 0)$</td>
</tr>
<tr>
<td>s2</td>
<td>start</td>
<td>v_accel</td>
<td>$v_{trail} &lt; v_{accel}(v_{ahead}, \Delta x_{ahead}) - (a_{max}(v_{trail})^2 - a_{com}^2)/2j_{com}$</td>
</tr>
<tr>
<td>d1</td>
<td>v_decel</td>
<td>v_accel</td>
<td>$v_{trail} &lt; v_{accel}(v_{ahead}, \Delta x_{ahead}) - (a_{max}(v_{trail})^2 - a_{com}^2)/2j_{com}$</td>
</tr>
<tr>
<td>d2</td>
<td>v_decel</td>
<td>threat</td>
<td>$v_{trail} &gt; v_{safe0}(v_{lead}, \Delta x)$</td>
</tr>
<tr>
<td>a1</td>
<td>v_accel</td>
<td>finish</td>
<td>$v_{trail} \geq v_{lead} - a_{trail}^2/2j_{com}$</td>
</tr>
<tr>
<td>a2</td>
<td>v_accel</td>
<td>threat</td>
<td>$v_{trail} &gt; v_{safe0}(v_{lead}, \Delta x)$</td>
</tr>
<tr>
<td>a3</td>
<td>v_accel</td>
<td>leader</td>
<td>$\Delta x \geq \Delta x_{desired}$</td>
</tr>
<tr>
<td>f1</td>
<td>finish</td>
<td>leader</td>
<td>$v_{trail} \geq v_{lead}$ or $v_{trail} \leq 0$</td>
</tr>
<tr>
<td>u1</td>
<td>threat</td>
<td>leader</td>
<td>$v_{trail} \leq v_{safe0}(v_{lead}, 0)$</td>
</tr>
</tbody>
</table>

$\Delta x_{ahead}$ for $t + (a_{max} + a_{com})/j_{com}$

Table 2: Transition table for state machine shown in Figure 5
A saturation function is assumed to keep the jerk within comfort levels when the lead platoon has jerk greater than or equal to zero. The saturation function is used for all states but threat.

- $v_{\text{decel}}$ Once the safety boundary has been crossed, maintaining comfort levels becomes more important. The desired acceleration in this state is:

\[
\ddot{x}_{\text{desired}} = -a_{\text{com}}. \tag{23}
\]

The control law is:

\[
u = \ddot{x} = k_a (\ddot{x}_{\text{desired}} - \ddot{x}) \tag{24}\]

Two transitions are possible from this state. If the lead platoon is decelerating at more than the comfort limit, the trail platoon will eventually cross the safety boundary and switch to the threat state. The more likely transition is to the $v_{\text{accel}}$ state. The condition for this transition is similar to those for entering the blend states in the merge maneuver. No blend state is necessary in the split maneuver because of the saturation function.

The transition to the $v_{\text{accel}}$ state requires an estimate of $a_{\text{max}}$. The estimated value of $a_{\text{max}}$ when the transition is taken is $a_{\text{maxs}}$, which is used in the control law for the next state.

- $v_{\text{accel}}$ Two possibilities exist for this state. In one case, the lead platoon is maintaining its velocity or accelerating. The desired acceleration under those circumstances is:

\[
\ddot{x}_{\text{desired}} = a_{\text{maxs}}. \tag{25}
\]

If the lead platoon is decelerating, however, the trail platoon must apply less acceleration to reach the desired spacing at the proper velocity. The desired acceleration in this case is:

\[
\ddot{x}_{\text{desired}} = \ddot{x}_{\text{lead}} + a_{\text{maxs}}. \tag{26}
\]

The desired acceleration is not allowed to drop below $-a_{\text{com}}$. The control law in this state is the same as above:

\[
u = \ddot{x} = k_a (\ddot{x}_{\text{desired}} - \ddot{x}) \tag{27}\]

Because acceleration capability drops as velocity increases, the trail platoon is not expected to maintain $a_{\text{maxs}}$. In that case, it takes the transition to the leader state. The platoon ends the maneuver while still accelerating because its velocity is less than that of the lead platoon. If calculating $a_{\text{max}}$ for a platoon is cumbersome, a fixed minimum acceleration could be substituted.

If the trail platoon’s velocity approaches that of the lead platoon, it takes the transition to finish, which reduces its acceleration.
Figure 6: Finite state machine model of controller for decelerate to change lanes maneuver. The states are listed in bold. The edge labels correspond to the conditions in table 3.

- **threat** In this state, the platoon is near an unsafe velocity. The controller seeks to move the platoon away from the safety boundary as quickly as possible. In the split, the trail platoon remains in this state until it is traveling appreciably slower than the lead platoon. Then the maneuver aborts.

  \[ u = \ddot{x} = -j_{\text{max}} \]  

- **finish** This state completes the maneuver. It reduces the trail platoon’s acceleration to zero by applying negative maximum comfortable jerk.

  \[ u = \ddot{x} = -j_{\text{com}} \]  

- **leader** This state is not actually part of the maneuver. It refers to the normal controller for a platoon leader described in (Godbole, Lygeros 1994).

### 5.2 Decelerate to Change Lanes

The state machine model of this maneuver is shown in figure 6. Transitions are listed in table 3.

This maneuver takes place between cars in separate lanes. \( v_{\text{lead}} \) refers to the velocity of the platoon in the adjacent lane. \( \Delta x \) is the longitudinal spacing between the platoon in the maneuver and the one in the adjacent lane. The control laws for the states in this maneuver are the same as those in the split maneuver. No \textit{start} or \textit{threat} states are needed because a collision between the two platoons in the maneuver can not occur. A separate control algorithm ensures that the platoon executing the maneuver does not collide with other platoons in its lane.

One new term has been added. \( v_{\text{min}} \) is a minimum safe velocity for the highway. When the trail platoon’s velocity falls below \( v_{\text{min}} \), a transition is taken from \( v_{\text{decel}} \) to \textit{leader}. This
addition accounts for the case in which the lead platoon is decelerating and the trail platoon can not reach a safe spacing without slowing to the point where it is a safety risk to the other platoons in its lane.

In certain situations the decelerate to change lanes maneuver may not involve any deceleration. Consider the case in which one platoon is close to the safe spacing and moving slower than the platoon ahead. The trail platoon must accelerate to match the velocity of the other platoon. As it does so, it will fall back to the safe spacing. The state machine is designed to handle this case. The controller would still start in the \( \nu_{decel} \) state, but would immediately switch into the \( \nu_{accel} \) state, which would take the platoon to the desired acceleration.

### 6 Simulations

The merge simulation results shown here are from a C program that simulates just the two platoons in the maneuver. The program was written to test the merge controller for any given lead platoon behavior. All three controllers were also tested in SmartPath (Eskafi, Varaiya 1992). The split results shown below are from a SmartPath simulation.

Parameter values for the simulations were set as follows:

- \( a_{com} = \pm 2 \text{ m/s}^2 \). This is the value used in the current merge (Godbole, Lygeros 1994). It is commonly accepted in the literature. See (Hitchcock 1994; Chiu et al. 1977).

- \( a_{min} = 5 \text{ m/s}^2 \). This is the absolute value of the maximum deceleration. This value is used in the current merge.

- \( a_{max} = 2.26 \text{ m/s}^2 - (0.05 \text{ s}^{-1})\nu_{trail} \). This linear function is a rough approximation based on data presented in (Gillespie 1992). The road is assumed to be flat. The vehicles are assumed to have automatic transmissions in third gear.

- \( j_{com} = \pm 2.5 \text{ m/s}^3 \). Lygeros and Godbole (Godbole, Lygeros 1994) set the comfortable jerk limit at \( 5 \text{m/s}^3 \) in the current merge. Most examples in the literature suggest the limit is between \( 2 \text{m/s}^3 \) and \( 2.5 \text{m/s}^3 \). See (Hitchcock 1994; Sklar et al. 1979; Chiu et al. 1977).
\( j_{\text{max}} = \pm 50 \text{ m/s}^3 \). This value was selected as a physical limit on jerk. It is less than the one given in (Fenton 1979).

\( v_{\text{allow}} = 3 \text{ m/s} \). The severity of injuries in automobile accidents is measured on the Abbreviated Injury Scale (AIS). Injuries rated from 3 to 6 on this scale are considered serious. Injuries of AIS = 2 are moderate. They are not life threatening but may be temporarily incapacitating. Examples are simple bone fractures or major abrasions (AAAM 1980). Fatalities are not measured on this scale. Using actual crash data, Hitchcock related AIS values to relative velocity at impact (Hitchcock 1993). For crashes at or below 3.3 m/s, he found no probability of fatalities or injuries rated AIS \( \geq 3 \). The probability of injuries rated AIS = 2 at that speed or slower is low.

\( \Delta x_{\text{desired}} = 1 \text{ m} \). This is the current intraplatoon spacing.

\( d = 20 \text{ msec} \). Simple brake models often include pure time delays of about 50 msec. It is shown in (Gerdes et al. 1993), however, that delays in the current braking system for PATH are greater than 150 msec. By redesigning the brake system, delays near 20 msec could be achieved (Gerdes, Maciuca 1994). Delays from sensing, filtering and differentiating are also possible, but they could be small at a high sample rate. The sample time used in the simulations shown here was 20 msec.

\( k_a = 8, k_v = 16 \), and the \( v_{\text{buffer}} = 0.31 \text{ m/s} \). These values were determined by examining the error behavior of the controller in the presence of disturbances. The crucial control law for determining \( v_{\text{buffer}} \) is the one for the first section of the safe velocity curve. The error dynamics for that law are:

\[
e = \frac{M}{s^2 + k_a s + k_v},
\]

where \( e = \dot{x}_{\text{trail}} - \dot{x}_{\text{desired}} \) and \( M \) is the upper bound on the sum of all terms in the control law that involve disturbances.

The disturbances considered were errors in calculating \( a_{\text{lead}} \) and \( \dot{j}_{\text{lead}} \) and errors in sensing \( v_{\text{lead}} \) and \( \Delta x \). Because a filter is assumed to reduce noise, the error considered here is any persistent bias in the signal.

The bounds on truncation errors from numerical differentiation are denoted \( E_a \) and \( E_j \). The differencing formulas are given in Appendix C. Including only the lowest order error terms, the error bounds are:

\[
E_a = T^2 \frac{\dot{j}_{\text{max}}}{6}
\]

and

\[
E_j = T \frac{\ddot{j}_{\text{max}}}{6}.
\]
where $T$ is the step size and $j_{lead}^{max}$ is the maximum derivative of the lead platoon's jerk. The sensor errors are denoted $\delta v$ and $\delta \Delta x$. They are used to define the quantity $E_v$:

$$E_v = \sqrt{(v_{lead} + \delta v)^2 + v_{allow}^2 + 2a_{min}(\Delta x + \delta \Delta x)} - \sqrt{v_{lead}^2 + v_{allow}^2 + 2a_{min}\Delta x}$$

(33)

$M$ is a function of the full state of the lead platoon and $E_a, E_j, E_v$ and $\delta v$. It can be simplified by the following assumptions:

1. $1/v_{lead} \approx 1/(v_{lead} + E_v)$
2. $v_{lead} \geq |v_{lead} - v_{trail}|$
3. $E_a\delta v \approx (E_j\delta v)/v_{lead} \approx 0$
4. $|a_{lead}| \leq a_{min}$
5. $|j_{lead}| \leq j_{max}$
6. $v_{lead} \leq \sqrt{(v_{lead} + \delta v)^2 + v_{allow}^2 + 2a_{min}(\Delta x + \delta \Delta x)}$

The upper bound on all terms involving disturbances is then

$$M = \left(\frac{2k_ua_{min} + j_{max} + 8a_{min}^2/v_{lead}}{v_{lead}}\right)\delta v + \left(k_u + \frac{7a_{min}}{v_{lead}}\right)E_a + k_vE_v + E_j$$

(34)

Substituting equation (34) into equation (30) and applying a unit step input yields the velocity error under the assumed worst condition. This error is the required $v_{buffer}$ for the selected values of $k_u$ and $k_v$.

The sensor errors were taken as $\delta v = .1 m/s$ and $\delta \Delta x = .5 m$, and $j_{lead}^{max}$ was assumed to be $100 m/s^4$. The maximum error is then $.31 m/s$ for $k_u = 8$ and $k_v = 16$. Realistic values of $\delta x$ and $v_{lead}$ were used in calculating $M$.

Figure 7 shows results for a merge from 30 m initial spacing. The velocity of the platoon ahead was constant at 25 m/s. The maneuver was completed in 12 sec. Jerk and acceleration comfort limits were not exceeded. The final relative acceleration and velocity should not be zero. The simulation only ran to the point when the follower law takes effect.

Figure 8 shows results from a merge with an initial spacing of 60 m. The lead platoon maintained a constant velocity. The merge took 18 sec in this case.

Figures 9 through 11 show cases in which the lead platoon applies maximum braking while the trail platoon is in the first blend (figure 9), the first section of the safe velocity curve (figure 10) and the second section of the safe curve (figure 11). The simulations were allowed to run until the trail platoon either stopped or collided. Note that only the third simulation resulted in a collision, and the impact speed was lower than $v_{allow}$. The platoons may collide if the lead platoon brakes while the trail platoon is on the first section of the safe curve, but because of the buffer, the impact velocities are well under $v_{allow}$. 

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Figure 7: Simulation results of merge from 30 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. In the graphs, spacing refers to $\Delta z$, and rel. velocity is $v_{trail} - v_{lead}$.

Figure 8: Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s.
Figure 9: Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied maximum braking at 7.6 sec.

Figure 10: Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied maximum braking at 10.0 sec.
Figure 11: Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied maximum braking at 15.0 sec.

These figures include large spikes in jerk. The controller is designed so that comfort limits are disregarded when safety becomes critical. In these cases, the saturation function on jerk was overridden once the large lead platoon deceleration was detected.

In the final merge simulation, the lead platoon braked at the comfortable jerk level to the comfortable deceleration. No collision occurred. The results are shown in figure 12.

Simulations were also run with a delay that reflects the current brake arrangement. Figure 13 shows how the delay and allowed impact velocity affected the time to complete a merge.

The split maneuver was also simulated. Figure 14 shows the results of a split to 35 m spacing.

7 Conclusions

The control strategy proposed in this report improves the safety and comfort of the longitudinal platoon maneuvers by eliminating the timed trajectories used in other designs. The strategy also increases the robustness of the maneuvers to variable acceleration performance. By changing certain parameters in the control equations in cases of bad weather or road conditions, the maneuvers can also be made robust to variable deceleration performance.

The merge maneuver reduces the threat of high-speed collisions occurring during a merge. The major cost of the added safety is the extra time that the maneuver takes to complete, particularly when current braking delays are taken into account. The cost of ensuring a comfortable ride during the split and decelerate to change lanes maneuvers is also time, but
Figure 12: Simulation results of merge from 60 m initial spacing: The initial velocity of both lead and trail platoons was 25 m/s. The lead platoon applied comfort level braking at 15.0 sec.

Figure 13: Length of merge maneuver for various delays and allowed impact velocities: The initial velocity of both lead and trail platoons was 25 m/s. The initial spacing was 60 m.
Figure 14: Simulation results of split to 35 m spacing: The initial velocity of the original platoon was 25 m/s.

to a lesser degree than for the merge. A less critical cost is the additional control energy needed in the safe merge as compared to in the current maneuver. Increases in the time needed to complete these maneuvers reduce the time available for other maneuvers and ultimately limit the capacity of the automated highway (Hall 1993). If these maneuvers cause large reductions in capacity, alternatives to merging and splitting platoons will have to be considered more carefully.

Further work on the maneuvers is needed. The controller should be tested with feedback linearization on a realistic car model. Inputs that accurately reflect sensor readings also must be added to the model. The control strategy must also be tested and perhaps modified to handle degraded sensor operation or sensor failure.

A switching controller similar to those described in this paper could also control cars in the normal leader mode. A leader controller is currently being designed.

References


A Calculations

A.1 Safe Velocity Curve Derivation

To find the safe velocity curve given by (2), the emergency jerk of both platoons was assumed to be large. The lead platoon could reach maximum deceleration essentially immediately, and after the pure time delay, so could the trail platoon. \( a_{\text{min}} \) is the absolute value of the maximum deceleration. The equations of motion are:

\[
x_{\text{lead}}(t) = \Delta x + v_{0\text{lead}}t - a_{\text{min}}t^2/2
\]

\[
v_{\text{lead}}(t) = v_{0\text{lead}} - a_{\text{min}}t
\]

\[
x_{\text{trail}}(t) = v_{0\text{trail}}t - a_{\text{min}}(t - d)^2/2
\]

\[
v_{\text{trail}}(t) = v_{0\text{trail}} - a_{\text{min}}(t - d)
\]

\( v_{0\text{lead}} \) and \( v_{0\text{trail}} \) are the velocities of the lead and trail platoons at the moment the lead platoon decelerates.

We now determine the position of the lead platoon when it stops.

\[
t_{\text{stop}} = v_{0\text{lead}}/a_{\text{min}}
\]

\[
x_{\text{lead}}(t_{\text{stop}}) = \Delta x + v_{0\text{lead}}^2/2a_{\text{min}}
\]

Next we determine when the trail platoon reaches \( v_{\text{allow}} \). At this time the platoons should collide.

\[
t_{\text{hit}} = (v_{0\text{trail}} + a_{\text{min}}d - v_{\text{allow}})/a_{\text{min}}
\]

\[
x_{\text{trail}}(t_{\text{hit}}) = v_{0\text{trail}}(v_{0\text{trail}} + a_{\text{min}}d - v_{\text{allow}})/a_{\text{min}} - (v_{0\text{trail}} - v_{\text{allow}})^2/2a_{\text{min}}
\]

Equating \( x_{\text{lead}}(t_{\text{stop}}) \) and \( x_{\text{trail}}(t_{\text{hit}}) \) and simplifying gives:

\[
(v_{0\text{trail}} + a_{\text{min}}d)^2 = v_{0\text{lead}}^2 + v_{\text{allow}}^2 + 2a_{\text{min}}\Delta x + (a_{\text{min}}d)^2
\]

Assuming that the delay is small and the final term can be ignored, this equation can be written:

\[
v_{\text{saf}} = -a_{\text{min}}d + \sqrt{v_{0\text{lead}}^2 + v_{\text{allow}}^2 + 2a_{\text{min}}\Delta x}
\]

If the initial assumption of a large jerk is not made, and \( v_{\text{allow}} = 0 \), the equation becomes

\[
v_{\text{saf}} = -a_{\text{min}}d + \sqrt{v_{0\text{lead}}^2 + (v_{\text{lead}} - v_{\text{trail}})a_{\text{min}}^2/j_{\text{max}}^2 + 2a_{\text{min}}\Delta x + (a_{\text{min}}d)^2}
\]

For \( j_{\text{max}} = 50 \text{m}/\text{s}^3 \), the extra term is not significant. This equation can be put in the same form as those in (Sklar et al. 1979). The authors of (Sklar et al. 1979) used similar techniques to determine safe following distances for non-emergency situations.
A.2 Deceleration Curve Derivation

At the end of the deceleration curve, the trail platoon should use the maximum comfortable jerk to bring it to its final spacing, velocity and acceleration. The difference in acceleration between the lead and trail platoons before applying that jerk is $-a_{com}$, so the time required to reach zero acceleration is $a_{com}/j_{com}$. Assuming the final velocity is $v_{lead}$, the initial blend velocity is given by:

$$v_{trail} = v_{lead} + a_{com}^2/j_{com} - a_{com}^2/2j_{com} = v_{lead} + a_{com}^2/2j_{com}$$

The final spacing should be $\Delta x_{desired}$, so the initial blend spacing is given by:

$$\Delta x = \Delta x_{desired} + a_{com}^3/2j_{com} - a_{com}^3/2j_{com} + a_{com}^3/6j_{com}$$

$$\Delta x = \Delta x_{desired} + a_{com}^3/6j_{com}$$

The initial blend values are the final values for the deceleration curve. On the deceleration curve, comfort level deceleration is assumed when the lead platoon is maintaining a constant velocity. When the lead platoon is accelerating or decelerating, the acceleration difference between platoons is the comfort level. The equations of motion are:

$$x_{lead}(t) = \Delta x + v_{0lead}t + a_{lead}t^2/2$$

$$v_{lead}(t) = v_{0lead} + a_{lead}t$$

$$x_{trail}(t) = v_{0trail}t - (a_{com} - a_{lead})t^2/2$$

$$v_{trail}(t) = v_{0trail} - (a_{com} - a_{lead})t$$

$$\Delta v_{final} = -a_{com}^2/2j_{com} = \Delta v_{initial} + a_{com}t_{final}$$

$$t_{final} = -(a_{com}^2/2j_{com} + \Delta v_{initial})/a_{com}$$

$$\Delta x_{final} = \Delta x_{desired} + a_{com}^3/6j_{com}^2 = x_{lead}(t_{final}) - x_{trail}(t_{final})$$

Solving this equation for $v_{0trail}$ gives

$$v_{0trail} = v_{0lead} + \sqrt{a_{com}^4/4j_{com}^2 + 2a_{com}(\Delta x - \Delta x_{desired} - a_{com}^3/6j_{com}^2)}$$

The resulting equation can be rewritten

$$v_{decel} = v_{lead} + \sqrt{a_{com}^4/4j_{com}^2 + 2a_{com}(\Delta x - \Delta x_{desired} - a_{com}^3/6j_{com}^2)}$$

This is equation (6).
B Desired Jerk and Acceleration Terms

The desired jerk and acceleration terms used in the control laws are given below:

- Deceleration curve:
  \[
  \ddot{x}_{\text{desired}} = \ddot{x}_{\text{lead}} + \frac{a_{\text{com}}(v_{\text{lead}} - v_{\text{trail}})}{\sqrt{a_{\text{com}}^4/4 j_{\text{com}}^2 + 2a_{\text{com}}(\Delta x - \Delta x_{\text{desired}} - a_{\text{com}}^3/6 j_{\text{com}}^2)}}
  \]

  \[
  \ddot{x}_{\text{desired}} = \ddot{j}_{\text{lead}} - \frac{a_{\text{com}}(v_{\text{lead}} - v_{\text{trail}})^2}{(\sqrt{a_{\text{com}}^4/4 j_{\text{com}}^2 + 2a_{\text{com}}(\Delta x - \Delta x_{\text{desired}} - a_{\text{com}}^3/6 j_{\text{com}}^2)})^3} + \frac{a_{\text{com}}(a_{\text{lead}} - a_{\text{trail}})}{\sqrt{a_{\text{com}}^4/4 j_{\text{com}}^2 + 2a_{\text{com}}(\Delta x - \Delta x_{\text{desired}} - a_{\text{com}}^3/6 j_{\text{com}}^2)}}
  \]

  where \(a_{\text{lead}}\) and \(\dot{j}_{\text{lead}}\) are the acceleration and jerk of the lead platoon estimated by numerical differentiation. \(a_{\text{trail}}\) is the acceleration of the trail platoon, which is assumed available from a sensor.

- First section of safe velocity curve:
  \[
  \ddot{x}_{\text{desired}} = \frac{v_{\text{lead}} a_{\text{lead}} + a_{\text{min}}(v_{\text{lead}} - v_{\text{trail}})}{\sqrt{v_{\text{lead}}^2 + v_{\text{allow}}^2 + 2a_{\text{min}}\Delta x}}
  \]

  \[
  \ddot{x}_{\text{desired}} = \frac{v_{\text{lead}} \dot{j}_{\text{lead}} + a_{\text{lead}}(a_{\text{lead}} + a_{\text{min}}) - a_{\text{min}} a_{\text{trail}}}{\sqrt{v_{\text{lead}}^2 + v_{\text{allow}}^2 + 2a_{\text{min}}\Delta x}} - \frac{(a_{\text{lead}}v_{\text{lead}} + a_{\text{min}}(v_{\text{lead}} - v_{\text{trail}}))^2}{(\sqrt{v_{\text{lead}}^2 + v_{\text{allow}}^2 + 2a_{\text{min}}\Delta x})^3}
  \]

- Second section of safe velocity curve:
  \[
  \ddot{x}_{\text{desired}} = a_{\text{lead}}
  \]

  \[
  \ddot{x}_{\text{desired}} = \ddot{j}_{\text{lead}}
  \]

C Differencing equations

The acceleration and jerk of the lead platoon are calculated numerically. In the current simulation, the following backward differencing equations are used:

\[
a_{\text{lead}}(k) = \frac{3v_{\text{lead}}(k) - 4v_{\text{lead}}(k - 1) + v_{\text{lead}}(k - 2)}{2T}
\]

\[
\dot{j}_{\text{lead}}(k) = \frac{v_{\text{lead}}(k) - 2v_{\text{lead}}(k - 1) + v_{\text{lead}}(k - 2)}{T^2},
\]

where \(T\) is the time step and \(k\) is the time step index.
D  Glossary

All symbols used in this report are listed below:

- $a_{\text{min}}$: the absolute value of the maximum deceleration
- $a_{\text{max}}(v)$: the maximum attainable acceleration for a given velocity
- $a_{\text{maxs}}$: the estimated maximum attainable acceleration of the trail platoon when it switches from deceleration to acceleration in a split or decelerate to change lanes maneuver
- $a_{\text{com}}$: the absolute value of the maximum comfortable acceleration or deceleration
- $a_{\text{lead}}$: the acceleration of the lead platoon
- $a_{\text{trail}}$: the acceleration of the trail platoon
- $d$: the delay between the time when the lead platoon applies braking and the time when the trail platoon applies braking
- $\Delta x$: relative spacing between two platoons involved in a maneuver
- $\Delta x_{\text{desired}}(t)$: desired spacing trajectory from previous merge design
- $\Delta x_{\text{desired}}$: the desired intraplatoon spacing used in the new merge, and the desired interplatoon spacing in the split and decelerate to change lanes maneuvers
- $\Delta x_{\text{ahead1}}, \Delta x_{\text{ahead2}}, \Delta x_{\text{ahead3}}$: the expected relative spacing after a given time interval
- $\delta v$: error in relative velocity sensor
- $\delta \Delta x$: error in relative spacing sensor
- $e$: the error between desired and actual velocity of the trail platoon
- $E_a$: truncation error from numerical calculation of lead platoon acceleration
- $E_j$: truncation error from numerical calculation of lead platoon jerk
- $E_v$: difference between true desired velocity and desired velocity calculated from inaccurate sensors
- $j_{\text{com}}$: the absolute value of the maximum comfortable jerk
- $j_{\text{max}}$: the absolute value of the maximum negative jerk possible in braking
- $j_{\text{lead}}$: the jerk of the lead platoon in a maneuver
• $j_{\text{lead}}^\text{max}$: maximum derivative of lead platoon jerk

• $k_a, k_v$: controller gains

• $M$: the maximum sum of all terms involving estimation or sensor errors in the control laws

• $T$: sample time of controller

• $v_{\text{safe1}}$: the velocity defined by the first portion of the maximum safe velocity curve

• $v_{\text{safe1}=+}$: the velocity defined by the first portion of the safe velocity curve minus a buffer velocity

• $v_{\text{safe2}}$: the velocity defined by the second portion of the maximum safe velocity curve

• $v_{\text{safe2}=+}$: the velocity defined by the second portion of the maximum safe velocity curve minus a buffer velocity

• $v_{\text{safe0}}$: the velocity defined by the maximum safe velocity curve from the case when the allowable impact velocity is zero

• $v_{\text{decel}}$: the desired velocity profile for the deceleration portion of the merge

• $v_{\text{decel}=+}$: the modified velocity profile for the deceleration portion of the merge; the modified profile allows the trail platoon space to reach the proper acceleration and spacing simultaneously

• $v_{\text{accel}}$: the desired velocity profile for the acceleration portion of the merge

• $v_{\text{allow}}$: the allowable impact velocity

• $v_{\text{buffer}}$: the velocity subtracted from the true maximum safe velocity curve to give the curve that the merge controller tracks

• $v_{\text{lead}}$: the velocity of the lead platoon in a maneuver

• $v_{\text{trail}}$: the velocity of the trail platoon in a maneuver

• $v_{\text{ahead}}$: the expected velocity of the trail platoon after a given time interval

• $v_{\text{ahead1}}, v_{\text{ahead2}}, v_{\text{ahead3}}$: the expected velocity of the lead platoon after a given time interval

• $v_{\text{accel}}$: the surface that defines when the split and decelerate to change lanes controllers should switch from deceleration to acceleration

• $v_{\text{min}}$: the minimum allowable velocity on the highway
• $u$: control output, assumed equal to the desired jerk

• $u_1$: jerk from feedback laws in blends in merge controller

• $\ddot{x}$: the jerk given by the control laws

• $\ddot{x}_{desired}$: the desired jerk from the velocity profiles